521317S, Wireless Communications II

Final exam 6 June 2015. Leave a margin of two columns at the right-hand side of each page. Mark clearly where a solution to a problem ends and if it continues on a following page or paper. Use of pencil in the solutions is allowed.

1. Consider a fast fading K-user uplink channel

$$y[m] = \sum_{k=1}^{K} h_k[m]x_k[m] + w[m] \tag{1}$$

where $h_k[m]$ is the normalised channel coefficient of user k at time instant m, $x_k[m]$ is the TX symbol of user k at time instant m, subject to $\mathbb{E}[|x_k|^2] \leq P_k$ and $w[m] \sim \mathcal{CN}(0, N_0)$ is i.i.d. complex Gaussian noise.

1.1 Write and depict the ergodic capacity region for 2-user ($K = 2$) *fast fading* channel assuming *full channel state information (CSI)* knowledge both at the transmitters and the receiver. Highlight the sum rate optimal boundary point and describe a simple power allocation scheme that can achieve the point. Explain how weighted sum rate maximization can be used to find all points on the boundary of the capacity region.

1.2 For $K = 3$ and with *CSI at the receiver only*, derive the ergodic sum rate capacity via mutual information. Assuming decoding order $3, 2, 1$, write the ergodic user specific rate expression $R_k, k = 1, 2, 3$. *Hint: Calculate first the rates conditioned on a single channel realisation, and then take the average over the fading distribution.* $I(x_1, x_2, x_3; y) = h(y) - h(y|x_1, x_2, x_3) = I(x_1; y) + I(x_2; y|x_1) + I(x_3; y|x_1, x_2)$.

2. Assume time-invariant uplink channel with a single BS with n_r receive antennas and K users, where each user k is equipped with n_{t_k} transmit antennas. The received signal vector at symbol time m is

$$\mathbf{y}[m] = \sum_{k=1}^{K} \mathbf{H}_k\mathbf{x}_k[m] + \mathbf{n}[m] \tag{2}$$

where $\mathbf{x}_k[m]$ is the TX vector of user k at time instant m, subject to $\mathbb{E}[\mathrm{Tr}(\mathbf{x}_k\mathbf{x}_k^\mathrm{H})] = \mathrm{Tr}(\mathbf{K}_{x_k}) \leq P_k$, $\mathbf{y} \in \mathbb{C}^{n_\mathrm{r}}$ is the RX signal, $\mathbf{w} \sim \mathcal{CN}(0, N_0\mathbf{I})$ is complex white Gaussian noise, and $\mathbf{H}_k \in \mathbb{C}^{n_\mathrm{r}\times n_{\mathrm{t}_k}}$ is the channel matrix of user k.

2.1 Draw a figure illustrating the system model in (2).

2.2 Assuming $n_{\mathrm{t}_k} = 1 \;\forall\; k$ and $\mathbf{H}_k = \mathbf{h}_k \in \mathbb{C}^{n_\mathrm{r}}$, show that MMSE-SIC is the capacity achieving receiver architecture, i.e.,

$$\sum_{i=1}^{K} \log(1 + \gamma_i^{\mathrm{mmse-sic}}) = \log\det(\mathbf{I}_{n_\mathrm{r}} + \sum_{i=1}^{K} \frac{P_i}{N_0}\mathbf{h}_i\mathbf{h}_i^\mathrm{H}) \tag{3}$$

where $\gamma_i^{\mathrm{mmse-sic}}$ is the SINR of the user i at the output of the MMSE-SIC receiver. *Hint: for* $\mathbf{a} \in \mathbb{C}^{m\times 1}$, $\mathbf{b} \in \mathbb{C}^{1\times m}$, *and invertible* $\mathbf{R} \in \mathbb{C}^{m\times m}$, $\log\det(\mathbf{R} + \mathbf{ab}) = \log\det(\mathbf{R}) + \log(1 + \mathbf{b}\mathbf{R}^{-1}\mathbf{a})$.

2.3 For $K = 4$, let $\{n_{t_1}, n_{t_2}, n_{t_3}, n_{t_4}\} = \{1, 2, 2, 2\}$, $n_r = 6$ and $P_k = P, \forall\, k$. What are the possible stream allocation alternatives (streams with non-zero power depending on the channel realisations $\mathbf{H}_k \;\forall\, k$) per user at high SNR? Assume *full CSIT* knowledge at all nodes. Justify your answer.

3. Consider time-invariant downlink channel with 3 single-antenna users and a single BS with n_t transmit antennas. The received signal vector $y_k \in \mathbb{C}$ for user k at symbol time m is described by

$$y_k[m] = \sum_{i=1}^{3} \mathbf{h}_k^{\mathrm{H}} \mathbf{u}_i x_i[m] + w_k[m] \tag{4}$$

where $x_k = \sqrt{p_k} d_k$ is the TX symbol of user k split into the normalised data symbol $d_k \in \mathbb{C}$ $(\mathrm{E}[|d_k|^2] = 1)$ and the corresponding power allocation p_k, $\mathbf{u}_k \in \mathbb{C}^{n_t}$ is the normalised beamformer, $\|\mathbf{u}_k\| = 1$, $w_k \sim \mathcal{CN}(0, N_0)$ is the complex white Gaussian noise and $\mathbf{h}_k \in \mathbb{C}^{n_t}$ is the channel vector of user k ideally known at the transmitter.

3.1 Draw a figure illustrating the system model in (4).

3.2 Write the signal-to-interference-plus-noise ratio (SINR) of user k assuming linear beamforming.

3.3 Write the SINR of user 2 assuming Costa (dirty paper) precoding and encoding order 1,3,2.

3.4 Assume the channels are orthogonal, i.e., $\mathbf{h}_k^{\mathrm{H}} \mathbf{h}_i = 0 \;\forall\, i \neq k$. What is the optimal SINR maximising beamformer $\mathbf{u}_k, \forall\, k$? Justify your answer.

3.5 For fixed *linear* beamformers $\mathbf{u}_k, k = 1, \ldots, 3$ and equal target SINR per user γ_{target}, write the single matrix expression for finding the optimal power allocation $\mathbf{p} = [p_1, p_2, p_3]^{\mathrm{T}}$.