521317S, Wireless Communications II

Final exam 7 June 2014. Leave a margin of two columns at the right-hand side of each page. Mark clearly where a solution to a problem ends and if it continues on a following page or paper. Use of pencil in the solutions is allowed.

1. Consider OFDMA transmission on 2-user time-invariant frequency-selective uplink channel with n_c sub-carriers. On the cth sub-carrier the received signal is written as

$$y_c = \sum_{k=1}^{2} h_{k,c} x_{k,c} + w_c \tag{1}$$

where $h_{k,c}$ is the frequency domain channel coefficient of user k, $x_{k,c}$ is the TX symbol of user k, subject to $\sum_{c=1}^{n_\text{c}} [|x_{k,c}|^2] \leq P_k$ and $w_c \sim \mathcal{CN}(0, N_0)$ is i.i.d. complex Gaussian noise

1.1 Write and depict the capacity region assuming full knowledge of the channels. When is the orthogonal allocation optimal (assuming n_c is large)?

1.2 Formulate an optimisation problem which maximise the weighted sum of user rates (WSRM) over the n_c sub-carriers.

1.3 Sketch an iterative algorithm to find rate point $R_1 = 2R_2$ on the boundary of the rate region. *Hint:* Adapt the weights in the WSRM algorithm to find the desired rate pair.

2. Assume time-invariant uplink channel with a single BS with n_r receive antennas and K users, where each user k is equipped with n_{t_k} transmit antennas. The received signal vector at symbol time m is

$$\mathbf{y}[m] = \sum_{k=1}^{K} \mathbf{H}_k \mathbf{x}_k[m] + \mathbf{n}[m] \tag{2}$$

where $\mathbf{x}_k[m]$ is the TX vector of user k at time instant m, subject to $\mathbb{E}[\text{Tr}(\mathbf{x}_k \mathbf{x}_k^\text{H})] = \text{Tr}(\mathbf{K}_{x_k}) \leq P_k$, $\mathbf{y} \in \mathbb{C}^{n_\text{r}}$ is the RX signal, $\mathbf{w} \sim \mathcal{CN}(0, N_0 \mathbf{I})$ is complex white Gaussian noise, and $\mathbf{H}_k \in \mathbb{C}^{n_\text{r} \times n_{\text{t}_k}}$ is the channel matrix of user k.

2.1 Draw a figure illustrating the system model in (2).

2.2 Assuming $n_{\text{t}_k} = 1 \;\forall\; k$ and $\mathbf{H}_k = \mathbf{h}_k \in \mathbb{C}^{n_\text{r}}$, show that MMSE-SIC is the capacity achieving receiver architecture, i.e.,

$$\sum_{i=1}^{K} \log(1 + \gamma_i^{\text{mmse}-\text{sic}}) = \log\det(\mathbf{I}_{n_\text{r}} + \sum_{i=1}^{K} \frac{P_i}{N_0} \mathbf{h}_i \mathbf{h}_i^\text{H}) \tag{3}$$

where $\gamma_i^{\text{mmse}-\text{sic}}$ is the SINR of the user i at the output of the MMSE-SIC receiver. *Hint: for* $\mathbf{a} \in \mathbb{C}^{m \times 1}$, $\mathbf{b} \in \mathbb{C}^{1 \times m}$, *and invertible* $\mathbf{R} \in \mathbb{C}^{m \times m}$, $\log\det(\mathbf{R} + \mathbf{ab}) = \log\det(\mathbf{R}) + \log(1 + \mathbf{b}\mathbf{R}^{-1}\mathbf{a})$.

2.3 For $K = 4$, let $\{n_{\text{t}_1}, n_{\text{t}_2}, n_{\text{t}_3}, n_{\text{t}_4}\} = \{1, 2, 2, 2\}$, $n_\text{r} = 6$ and $P_k = P, \forall\; k$. What are the possible stream allocation alternatives (streams with non-zero power depending on the channel realisations $\mathbf{H}_k \;\forall\; k$) per user at high SNR? Assume *full CSIT* knowledge at all nodes. Justify your answer.

3. Consider time-invariant downlink channel with 3 single-antenna users and a single BS with n_t transmit antennas. The received signal vector $y_k \in \mathbb{C}$ for user k at symbol time m is described by

$$y_k[m] = \sum_{i=1}^{3} \mathbf{h}_k^\mathrm{H} \mathbf{u}_i x_i[m] + w_k[m] \tag{4}$$

where $x_k = \sqrt{p_k} d_k$ is the TX symbol of user k split into the normalised data symbol $d_k \in \mathbb{C}$ ($\mathrm{E}\left[|d_k|^2\right] = 1$) and the corresponding power allocation p_k, $\mathbf{u}_k \in \mathbb{C}^{n_\mathrm{t}}$ is the normalised beamformer, $\|\mathbf{u}_k\| = 1$, $w_k \sim \mathcal{CN}(0, N_0)$ is the complex white Gaussian noise and $\mathbf{h}_k \in \mathbb{C}^{n_\mathrm{t}}$ is the channel vector of user k ideally known at the transmitter.

3.1 Draw a figure illustrating the system model in (4).

3.2 Write the signal-to-interference-plus-noise ratio (SINR) of user k assuming linear beamforming.

3.3 Write the SINR of user 2 assuming Costa (dirty paper) precoding and encoding order 1,3,2.

3.4 Assume the channels are orthogonal, i.e., $\mathbf{h}_k^\mathrm{H} \mathbf{h}_i = 0 \; \forall \; i \neq k$. What is the optimal SINR maximising beamformer $\mathbf{u}_k, \forall \; k$? Justify your answer.

3.5 For fixed *linear* beamformers $\mathbf{u}_k, k = 1, \ldots, 3$ and equal target SINR per user γ_target, write the single matrix expression for finding the optimal power allocation $\mathbf{p} = [p_1, p_2, p_3]^\mathrm{T}$.