521317S, Wireless Communications III

Exam 13 September 2013. Leave a margin of two columns at the right-hand side of each page. Mark clearly where a solution to a problem ends and if it continues on a following page or paper. Use of pencil in the solutions is allowed.

1. Consider a fast fading K-user uplink channel

$$y[m] = \sum_{k=1}^{K} h_k[m]x_k[m] + w[m] \tag{1}$$

where $h_k[m]$ is the normalised channel coefficient of user k at time instant m, $x_k[m]$ is the TX symbol of user k at time instant m, subject to $\mathbb{E}[|x_k|^2] \leq P_k$ and $w[m] \sim \mathcal{CN}(0, N_0)$ is i.i.d. complex Gaussian noise.

1.1 Write and depict the ergodic capacity region for 2-user ($K = 2$) fast fading channel assuming *full channel state information (CSI)* knowledge both at the transmitters and the receiver. Highlight the sum rate optimal boundary point and describe a simple power allocation scheme that can achieve the point. Explain how weighted sum rate maximization can be used to find all points on the boundary of the capacity region.

1.2 For $K = 3$ and with *CSI at the receiver only*, derive the ergodic sum rate capacity via mutual information. Assuming decoding order $3, 2, 1$, write the ergodic user specific rate expression $R_k, k = 1, 2, 3$. *Hint: Calculate first the rates conditioned on a single channel realisation, and then take the average over the fading distribution.* $I(x_1, x_2, x_3; y) = h(y) - h(y|x_1, x_2, x_3) = I(x_1; y) + I(x_2; y|x_1) + I(x_3; y|x_1, x_2)$.

2. Assume time-invariant point-to-point MIMO channel with n_t transmit antennas and n_r receive antennas. The received signal vector at symbol time m is described by

$$\mathbf{y}[m] = \mathbf{H}\mathbf{x}[m] + \mathbf{w}[m] \tag{2}$$

where $\mathbf{x}$ is the transmit symbol vector of user k, subject to $\mathbb{E}[\mathrm{Tr}(\mathbf{x}\mathbf{x}^\mathrm{H})] = \mathrm{Tr}(\mathbf{K}_x) \leq P$, $\mathbf{y} \in \mathbb{C}^{n_\mathrm{r}}$ is the received signal, $\mathbf{w} \sim \mathcal{CN}(0, N_0\mathbf{I})$ is complex white Gaussian noise, and $\mathbf{H} \in \mathbb{C}^{n_\mathrm{r} \times n_\mathrm{t}}$ is the channel matrix

2.1 Derive the capacity of the system in (2) assuming Gaussian input distribution. Show that the general ($\log\det$) expression is equivalent to the one achieved via singular value decomposition of $\mathbf{H}$. *Hint:* $h(\mathbf{y}) \leq \log\det(\pi e \mathbb{E}[\mathbf{y}\mathbf{y}^\mathrm{H}])$.

2.2 Assuming CSI knowledge only at the receiver and $\mathbf{K}_x = \frac{P}{n_\mathrm{t}}\mathbf{I}_{n_\mathrm{t}}$, write the signal-to-interference-plus-noise ratio (SINR) per received data stream for the matched filter (MF), zero forcing (ZF), linear minimum mean square error (MMSE) and MMSE with successive interference cancellation (MMSE-SIC) receiver structures. Simplify the SINR expressions as much as possible. Compare (illustrate) their performance across the entire SNR range.

3. Consider time-invariant downlink channel with 2 single-antenna users and a single BS with n_t transmit antennas. The received signal vector $y_k \in \mathbb{C}$ for user k at symbol time m is described by

$$y_k[m] = \sum_{i=1}^{2} \mathbf{h}_k^\mathrm{H} \mathbf{u}_i x_i[m] + w_k[m] \tag{3}$$

where $x_k = \sqrt{p_k} d_k$ is the TX symbol of user k split into the normalised data symbol $d_k \in \mathbb{C}$ $(\mathrm{E}\left[|d_k|^2\right] = 1)$ and the corresponding power allocation p_k, $\mathbf{u}_k \in \mathbb{C}^{n_\mathrm{t}}$ is the normalised beamformer, $\|\mathbf{u}_k\| = 1$, $w_k \sim \mathcal{CN}(0, N_0)$ is the complex white Gaussian noise and $\mathbf{h}_k \in \mathbb{C}^{n_\mathrm{t}}$ is the channel vector of user k ideally known at the transmitter.

Assume the BS applies non-linear Costa precoding with user encoding order $1, 2$. Find the sum rate optimal precoders $\mathbf{m}_k = \sqrt{p_k}\mathbf{u}_k, k = 1, 2$, where $\mathbf{u}_k$ is the normalised transmit beamformer vector and p_k is the corresponding power allocation. *Hint: Start with dual uplink formulation with reverse decoding order, find the sum rate optimal powers and the MMSE-SIC receivers in the dual uplink, and apply the uplink-downlink duality to find the corresponding downlink precoders.*