521317S, Wireless Communications III

Exam 29 June 2013. Leave a margin of two columns at the right-hand side of each page. Mark clearly where a solution to a problem ends and if it continues on a following page or paper. Use of pencil in the solutions is allowed.

1. Consider a fading K-user uplink channel

$$y[m] = \sum_{k=1}^{K} h_k[m]x_k[m] + w[m] \tag{1}$$

where $h_k[m]$ is the normalised channel coefficient of user k at time instant m, $x_k[m]$ is the TX symbol of user k at time instant m, subject to $\mathbb{E}[|x_k|^2] \leq P_k$ and $w[m] \sim \mathcal{CN}(0, N_0)$ is i.i.d. complex Gaussian noise.

1.1 Assuming fast fading channel and $K = 1$, express the highest communication rate possible with channel state information (CSI) at the receiver only.

1.2 Assuming fast fading channel and $K = 1$, describe briefly the capacity optimal transmission strategy with the CSI knowledge both at the transmitter (CSIT) and the receiver. How does the capacity with full CSI compares to the AWGN channel ($h_k[m] = 1 \; \forall \; m$) and to the case without CSIT at low and high SNR?

1.3 Depict the capacity region for 2-user fast fading channel assuming that *only the receiver* has full knowledge of the channels. Give the expression for the achievable ergodic rate $R_k, k = 1, 2$ at each point in the boundary of the region. Identify scenarios where the point $R_1 = R_2$ is not sum rate optimal. Proof using Jensen's inequality that without CSIT the fading always hurts (as compared to the AWGN channel).

1.4 Assume slow fading, i.e., $h_k[m] = h_k \; \forall \; m$ and $K = 3$. Describe the *outage* probability for given rates $R_k, \; k = 1, 2, 3$.

2. Assume time-invariant point-to-point MIMO channel with n_t transmit antennas and n_r receive antennas. The received signal vector at symbol time m is described by

$$\mathbf{y}[m] = \mathbf{H}\mathbf{x}[m] + \mathbf{w}[m] \tag{2}$$

where $\mathbf{x}$ is the transmit symbol vector of user k, subject to $\mathbb{E}[\mathrm{Tr}(\mathbf{x}\mathbf{x}^\mathrm{H})] = \mathrm{Tr}(\mathbf{K}_x) \leq P$, $\mathbf{y} \in \mathbb{C}^{n_\mathrm{r}}$ is the received signal, $\mathbf{w} \sim \mathcal{CN}(0, N_0\mathbf{I})$ is complex white Gaussian noise, and $\mathbf{H} \in \mathbb{C}^{n_\mathrm{r} \times n_\mathrm{t}}$ is the channel matrix

2.1 Describe the capacity optimal linear transmission/reception scheme with full CSIT. Explain what happens to the number of active spatial streams both at low and high SNR. How does the scheme change when $\mathbf{w} \sim \mathcal{CN}(0, \mathbf{Z})$ and $\mathbf{Z}$ is also known at the transmitter?

2.2 Assuming CSI knowledge only at the receiver and $\mathbf{K}_x = \frac{P}{n_\mathrm{t}}\mathbf{I}_{n_\mathrm{t}}$, describe the matched filter, zero forcing (ZF), linear minimum mean square error (MMSE) and MMSE with successive interference cancellation (MMSE-SIC) receiver structures and their corresponding signal-to-interference-plus-noise ratios (SINR) per received data stream. Compare (illustrate) their performance across the entire SNR range.

3. Assume time-invariant uplink channel with 3 users each with n_{t_k} antennas and a single BS with n_{r} receive antennas. The received signal vector at symbol time m is described by

$$\mathbf{y}[m] = \sum_{k=1}^{3} \mathbf{H}_k \mathbf{x}_k[m] + \mathbf{n}[m] \tag{3}$$

where $\mathbf{x}_k[m]$ is the TX vector of user k at time instant m, subject to $\mathbb{E}[\mathrm{Tr}(\mathbf{x}_k \mathbf{x}_k^{\mathrm{H}})] \leq P_k$, $\mathbf{y} \in \mathbb{C}^{n_{\mathrm{r}}}$ is the RX signal, $\mathbf{w} \sim \mathcal{CN}(0, N_0\mathbf{I})$ is complex white Gaussian noise, and $\mathbf{H}_k \in \mathbb{C}^{n_{\mathrm{r}} \times n_{\mathrm{t}_k}}$ is the channel matrix of user k.

3.1 Let $n_{\mathrm{t}_k} = 2, \forall\, k$ and $n_{\mathrm{r}} = 6$ and $P_k = P, \forall\, k$. How many streams with non-zero power are allocated per user both at low and high SNR? Justify your answer.

3.2 Derive the capacity region of the system in (3) assuming Gaussian input distribution. Direct answer without a derivation (via mutual information) does not give points. *Hint:* $h(\mathbf{y}) \leq \log\det(\pi e \mathbb{E}[\mathbf{y}\mathbf{y}^{\mathrm{H}}])$, $I(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3; \mathbf{y}) = h(\mathbf{y}) - h(\mathbf{y}|\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3) = I(\mathbf{x}_1; \mathbf{y}) + I(\mathbf{x}_2; \mathbf{y}|\mathbf{x}_1) + I(\mathbf{x}_3; \mathbf{y}|\mathbf{x}_1, \mathbf{x}_2)$

3.3 Assume decoding order $3, 2, 1$ at the BS, write the rate expression $R_k, k = 1, 2, 3$.