521317S, Wireless Communications III

Exam 10 May 2013. Leave a margin of two columns at the right-hand side of each page. Mark clearly where a solution to a problem ends and if it continues on a following page or paper. Use of pencil in the solutions is allowed.

1. Consider a fast fading K-user uplink channel

$$y[m] = \sum_{k=1}^{K} h_k[m] x_k[m] + w[m] \tag{1}$$

where $h_k[m]$ is the normalised channel coefficient of user k (each with stationary and ergodic fading process), x_k is the TX symbol of user k, subject to $\mathbb{E}[|x_k|^2] \leq P_k$ and $w[m] \sim \mathcal{CN}(0, N_0)$ is i.i.d. complex Gaussian noise.

1.1 Draw a figure illustrating the system model in (1).

1.2 For $K = 2$, express and depict the (ergodic) capacity region for the case with channel state information (CSI) *at the receiver only.*

1.3 For $K = 3$ and with CSI at the receiver only, derive the ergodic sum rate capacity via mutual information. Assuming decoding order $1, 2, 3$, write the ergodic user specific rate expression $R_k, k = 1, 2, 3$. *Hint: Calculate first the rates conditioned on a single channel realisation, and then take the average over the fading distribution.* $I(x_1, x_2, x_3; y) = h(y) - h(y|x_1, x_2, x_3) = I(x_1; y) + I(x_2; y|x_1) + I(x_3; y|x_1, x_2)$.

1.4 With full CSI and $P_k = P \; \forall \; k$, describe the simple *sum rate optimal* transmission strategy.

1.5 With full CSI and $P_k = P \; \forall \; k$, explain the effect of multiuser diversity gain on the sum rate capacity as the number of users K is increased.

2. Assume time-invariant point-to-point MIMO channel with n_t transmit antennas and n_r receive antennas. The received signal vector at symbol time m is described by

$$\mathbf{y}[m] = \mathbf{H}\mathbf{x}[m] + \mathbf{w}[m] \tag{2}$$

where $\mathbf{x}$ is the transmit symbol vector of user k, subject to $\mathbb{E}[\mathrm{Tr}(\mathbf{x}\mathbf{x}^\mathrm{H})] = \mathrm{Tr}(\mathbf{K}_x) \leq P$, $\mathbf{y} \in \mathbb{C}^{n_\mathrm{r}}$ is the received signal, $\mathbf{w} \sim \mathcal{CN}(0, \mathbf{Z})$ is complex *coloured* Gaussian noise, and $\mathbf{H} \in \mathbb{C}^{n_\mathrm{r} \times n_\mathrm{t}}$ is the channel matrix

2.1 Derive the mutual information $I(\mathbf{x}; \mathbf{y})$ of the system in (2) in terms of $\mathbf{K}_x$. What is the capacity achieving distribution of $\mathbf{x}$ that maximises $I(\mathbf{x}; \mathbf{y})$? *Hint:* $h(\mathbf{y}) \leq \log\det(\pi e \mathbb{E}[\mathbf{y}\mathbf{y}^H])$

2.2 Assume $\mathbf{K}_x = \mathbf{Q}\mathbf{P}\mathbf{Q}^\mathrm{H}$,where $\mathbf{Q}$ is a unitary precoder matrix and $\mathbf{P}$ is a diagonal power allocation matrix. What is the capacity optimal $\mathbf{Q}$ and $\mathbf{P}$ when both $\mathbf{H}$ and $\mathbf{Z}$ are perfectly known at the transmitter *Hint: Consider a whitened channel* $\tilde{\mathbf{H}}$ *for finding the optimal* $\mathbf{K}_x$.

3. Consider time-invariant downlink channel with 3 single-antenna users and a single BS with n_{t} transmit antennas. The received signal vector $y_k \in \mathbb{C}$ for user k at symbol time m is described by

$$y_k[m] = \sum_{i=1}^{3} \mathbf{h}_k^{\mathrm{H}} \mathbf{u}_i x_i[m] + w_k[m] \tag{3}$$

where $x_k = \sqrt{p_k} d_k$ is the TX symbol of user k split into the normalised data symbol $d_k \in \mathbb{C}$ ($\mathrm{E}\left[|d_k|^2\right] = 1$) and the corresponding power allocation p_k, $\mathbf{u}_k \in \mathbb{C}^{n_{\mathrm{t}}}$ is the normalised beamformer, $\|\mathbf{u}_k\| = 1$, $w_k \sim \mathcal{CN}(0, N_0)$ is the complex white Gaussian noise and $\mathbf{h}_k \in \mathbb{C}^{n_{\mathrm{t}}}$ is the channel vector of user k ideally known at the transmitter.

3.1 Draw a figure illustrating the system model in (3).

3.2 Write the signal-to-interference-plus-noise ratio (SINR) of user k assuming linear beamforming.

3.3 Write the SINR of user 2 assuming Costa (dirty paper) precoding and encoding order 1,3,2.

3.4 Assume the channels are orthogonal, i.e., $\mathbf{h}_k^{\mathrm{H}} \mathbf{h}_i = 0 \ \forall \ i \neq k$. What is the optimal SINR maximising beamformer $\mathbf{u}_k, \forall \ k$? Justify your answer.

3.5 For fixed linear beamformers $\mathbf{u}_k, k = 1, \ldots, 3$ and equal target SINR per user γ_{target}, write the single matrix expression for finding the optimal power allocation $\mathbf{p} = [p_1, p_2, p_3]^{\mathrm{T}}$.