Harjoitus 1

1. a) $A = A_1 \cup A_2 \cup A_3$

b) $B = A_1 \cap A_2 \cap A_3$

c) $C = A_1 \cap A_2 \cap \overline{A_3}$

d) $D = \overline{A_1} \cap \overline{A_2} \cap \overline{A_3}$

e) $E = (A_1 \cap A_2 \cap \overline{A_3}) \cup (A_1 \cap A_3 \cap \overline{A_2}) \cup (A_2 \cap A_3 \cap \overline{A_1})$

2. a)$P[(A \cap B \cap \overline{C}) \cup (A \cap \overline{B} \cap C) \cup (\overline{A} \cap B \cap C)]$

$= P(A \cap B \cap \overline{C}) + P(A \cap \overline{B} \cap C) + P(\overline{A} \cap B \cap C)$ (tapahtumat erilliset)

$= P(A \cap B) + P(A \cap C) + P(B \cap C) - 3P(A \cap B \cap C)$.

b) Katso Venn-diagrammista tai

$P[(A \cap \overline{B} \cap \overline{C}) \cup (\overline{A} \cap B \cap \overline{C}) \cup (\overline{A} \cap \overline{B} \cap C)]$

$= P(A \cap (\overline{B \cup C})) + P(B \cap (\overline{A \cup C})) + P(C \cap (\overline{A \cup B}))$ (tap. erilliset + de Morgan)

$= P(A) - P(A \cap (B \cup C)) + P(B) - P(B \cap (A \cup C)) + P(C) - P(C \cap (A \cup B))$

$= P(A) - P((A \cap B) \cup (A \cap C)) + P(B) - P((B \cap A) \cup (B \cap C))$

$+ P(C) - P((C \cap A) \cup (C \cap B))$

$= P(A) - [P(A \cap B) + P(A \cap C) - P(A \cap B \cap C)] + P(B) - [P(B \cap A) + P(B \cap C) - P(A \cap B \cap C)] +$
$P(C) - [P(C \cap A) + P(C \cap B) - P(A \cap B \cap C)]$

$= P(A) + P(B) + P(C) - 2P(A \cap B) - 2P(A \cap C) - 2P(B \cap C) + 3P(A \cap B \cap C)$.

c) $P(\overline{A} \cap \overline{B} \cap \overline{C}) = P(\overline{A \cup B \cup C}) = 1 - P(A \cup B \cup C)$

$= 1 - P(A) - P(B) - P(C) + P(A \cap B) + P(A \cap C) + P(B \cap C) - P(A \cap B \cap C)$.

3. a) $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.7 + 0.6 - 0.4 = 0.9$

b) $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C) =$
$0.7 + 0.6 + 0.5 - 0.4 - 0.3 - 0.2 + 0.1 = 1$

c) $P(\overline{A} \cap \overline{B} \cap C) = P((\overline{A \cup B}) \cap C) = P(C) - P((A \cup B) \cap C)$

$= P(C) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C) = 0.5 - 0.3 - 0.2 + 0.1 = 0.1$

2. tapa: Katso Venn-diagrammista.

4. Seuraavassa kirjaimet a, b, c, d tarkoittavat keskenään erisuuria nopan silmälukuja. Käytetään klassista todennäköisyyttä:

Perusjoukon koko $|S| = 6 \cdot 6 \cdot 6 \cdot 6 \cdot 6$.

a) P(Yatzy)$=P$(kaikki samoja)$=$ $P(aaaaa) = \frac{6}{|S|} = \frac{1}{6^4}$.

b) P(täyskäsi)$=P$(kolmoset ja pari)$=$ P($aaabb$ kombinaatioineen, joita on 10 kpl) $= \frac{10 \cdot 6 \cdot 5}{|S|} = \frac{50}{6^4}$.

c) P(kolmoset tai pari, poislukien Yatzy, täyskäsi, neloset ja kaksi paria)

$=$ P($aaabc$ kombinaatioineen tai $aabcd$ kombinaatioineen)

$= \frac{10 \cdot 6 \cdot 5 \cdot 4 + 10 \cdot 6 \cdot 5 \cdot 4 \cdot 3}{|S|} = \frac{800}{6^4}$.

5. Perusjoukko

$S = \{(1,1,1),(1,1,0),(1,0,1),(0,1,1),(0,0,1),(0,1,0),(1,0,0),(0,0,0)\}$,

$|S| = 8$

$A = \{(1,1,0),(1,0,1),(0,1,1)\}$,

$|A| = 3$

$B = \{(1,1,0),(1,0,1),(0,1,1),(0,0,1),(0,1,0),(1,0,0),(0,0,0)\}$,

$|B| = 7$

$C = \{(1,0,1),(0,0,1),(0,1,0),(1,0,0),(0,0,0)\}$,

$|C| = 5$

$P(A) = \frac{|A|}{|S|} = \frac{3}{8}$

$P(B) = \frac{|B|}{|S|} = \frac{7}{8}$

$P(C) = \frac{|C|}{|S|} = \frac{5}{8}$