Statistical Signal Processing 1, Minor exam #1, 05-Oct-2023, 10:20-11:45

INSTRUCTIONS

Solve all three (3) problems. No material is allowed (no calculator etc.) except for two handwritten A4 papers (totally 4 A4 pages). Please make sure to return your solutions on time. Write your name and your student ID on all papers. Clearly indicate which solution you are solving. Intermediate steps need to be included (do not just give the answer). Simplify your solutions to the extent possible. Return the exam answer paper(s) to the invigilator before leaving the exam hall. Show the student card (or mobile student card on the mobile phone) or identity card (passport, ID card, or KELA card with a photo) to the exam invigilator. Remember to staple the exam answer papers together in the correct order.

A. Problem 1

We use estimator

$$\widehat{\theta} = \frac{1}{2N} \sum_{i=1}^{N} \left(z[i]^2 - 2 \right)$$

to estimate unknown parameter θ . In addition we know that $E\left[z^2\left[i\right]\right]=2\left(\theta+1\right)$ (where $z^2\left[i\right]=z[i]\times z[i]$). Prove that $E\left[\hat{\theta}\right]=\theta$. The samples $z[i], i=1,2,\cdots,N$ are not necessarily independent. Justify each step given this fact.

B. Problem 2

Random variables X and Y have the joint probability density function (PDF)

$$f_{XY}\left(x,y\right) = \left\{ \begin{array}{ll} \frac{x^2}{4} + \frac{y^2}{4} + \frac{xy}{6} & 0 \leq x \leq 1, 0 \leq y \leq 2\\ 0 & \text{otherwise} \end{array} \right.$$

Assume that $0 \le y \le 2$.

- Find conditional PDF of X given Y = y
- Find $P(X < \frac{1}{2}|Y = y)$
- Find the expected value of 2Y 1, i.e, E[2Y 1]. Give the solution as a rational number.

C. Problem 3

Assume that X_1 and X_2 are random variables with joint probability density function (PDF)

$$f_{X_1X_2}(x_1, x_2) = \exp(-x_1 - x_2), 0 < x_1 < \infty, 0 < x_2 < \infty$$

Let us consider transformation of random variables:

$$Y_1 = X_1 + X_2$$

and

$$Y_2 = \frac{X_1}{X_1 + X_2}$$

Find the joint PDF of Y_1 and Y_2 . Remember to find the validity region for the joint PDF. In addition to the equations, visually show the validity region of the joint PDF in the two dimensional xy-plane [draw a grid similar to the figure in the opposite side of this paper]



wiseuss our si

(e-, (e) 7 Ye and

to express unlimber jurgqueter if, in addition we know hill; is still a 216 + 1)4 on the Prove that E | Fig. - V. The samples | V. v. - 1, 2, ... A first not recessarily independent the large and still post of the samples | V. v. - 1, 2, ... A first not recessarily independent the large and the first of the samples | V. v. - 1, 2, ... A first not recessarily independent at the large and the first of the samples | V. v. - 1, 2, ... A first not recessarily independent at the large and the

A STATE OF THE PARTY OF THE PAR

(PDP) muriquit granet yeithering more than from (PDP)

1202012120 8+ 1+ 4 1 (0.2) vA

Assume that the section of

war I made X to ACF applications to the

(I and the Lange of Property

I are decognized value of \$1 - 1, i.e. h ix) - 1]. Give the solution as a month outless of

From and

Assessed that Assessed with the modern entrained with proposition of the second (PER)

Les us consider tour avenue of random visitable

white the state of the

and the same

the state of the s