# Statistical Signal Processing 1, Minor exam #1, 05-Oct-2023, 10:20-11:45

## INSTRUCTIONS

Solve all three (3) problems. No material is allowed (no calculator etc.) except for two handwritten A4 papers (totally 4 A4 pages). Please make sure to return your solutions on time. Write your name and your student ID on all papers. Clearly indicate which solution you are solving. Intermediate steps need to be included (do not just give the answer). Simplify your solutions to the extent possible. Return the exam answer paper(s) to the invigilator before leaving the exam hall. Show the student card (or mobile student card on the mobile phone) or identity card (passport, ID card, or KELA card with a photo) to the exam invigilator. Remember to staple the exam answer papers together in the correct order.

### A. *Problem 1*

We use estimator

$$\widehat{\theta} = \frac{1}{2N} \sum_{i=1}^{N} \left( z[i]^2 - 2 \right)$$

to estimate unknown parameter $\theta$. In addition we know that $E\left[z^2\left[i\right]\right] = 2\left(\theta + 1\right)$ (where $z^2\left[i\right] = z[i] \times z[i]$). Prove that $E\left[\hat{\theta}\right] = \theta$. The samples $z[i], i = 1, 2, \cdots, N$ are not necessarily independent. Justify each step given this fact.

### B. *Problem 2*

Random variables $X$ and $Y$ have the joint probability density function (PDF)

$$f_{XY}\left(x, y\right) = \begin{cases} \frac{x^2}{4} + \frac{y^2}{4} + \frac{xy}{6} & 0 \leq x \leq 1, 0 \leq y \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

Assume that $0 \leq y \leq 2$.

- Find conditional PDF of $X$ given $Y = y$
- Find $P\left(X < \frac{1}{2} | Y = y\right)$
- Find the expected value of $2Y - 1$, i.e, $E\left[2Y - 1\right]$. Give the solution as a rational number.

### C. *Problem 3*

Assume that $X_1$ and $X_2$ are random variables with joint probability density function (PDF)

$$f_{X_1 X_2}\left(x_1, x_2\right) = \exp\left(-x_1 - x_2\right), 0 < x_1 < \infty, 0 < x_2 < \infty$$

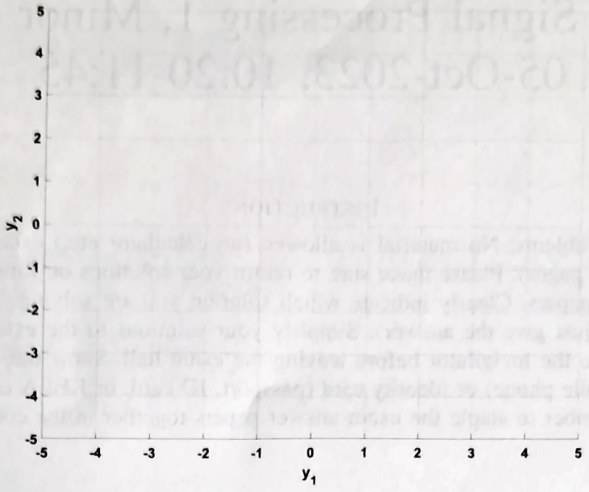
Let us consider transformation of random variables:

$$Y_1 = X_1 + X_2$$

and

$$Y_2 = \frac{X_1}{X_1 + X_2}$$

Find the joint PDF of $Y_1$ and $Y_2$. Remember to find the validity region for the joint PDF. In addition to the equations, visually show the validity region of the joint PDF in the two dimensional xy-plane [draw a grid similar to the figure in the opposite side of this paper]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