Continuous uniform distribution $U[a,b]$

$$f(x) = \frac{1}{b-a}, \quad a \leq x \leq b \qquad E[U[a,b]] = \frac{a+b}{2}$$

$$F(x) \begin{cases} 0 & , x < a \\ \frac{x-a}{b-a} & , a \leq x \leq b \\ & , x > b \end{cases} \qquad Var(U[a,b]) = \frac{(b-a)^2}{12}$$

Variance

$$Var(x) = \int_{\mathbb{R}} f(x)(x-\mu)^2 \, dx$$

Chi-Squared distribution χ^2

$$p(y) = \frac{1}{2^{N/2}\Gamma(N/2)} y^{(N/2)-1} e^{-y/2}$$

$$Var[x[n]] = E[x[n]^2] - E[x[n]]^2$$

$$\left(\sum_{i=0}^{N} f(i)\right)^2 = \sum_{i=0}^{N} \sum_{j=0}^{N} f(i) f(i)$$

$$Var(x[n]^2) = E[x[n]^4] - E[x[n]^2]^2$$

CRLB

$$= \frac{1}{-E\left\{\frac{\partial^2 \ln p(x;\theta)}{\partial \theta^2}\right\}} = \text{Thery of Signal in AWGN} = \frac{\sigma^2}{\sum_{n=0}^{N-1}\left(\frac{\partial \bar{x}[n]}{\partial A}\right)^2}$$

CRLB standard

Normal distribution

$$p(x) = \frac{1}{(2\pi\sigma^2)^{N/2}} \exp\left(-\frac{1}{2\sigma^2}\sum_{n=0}^{N-1}(x[n] - E[x[n]])^2\right)$$

$$\log p(x) = -\frac{N}{2}\log(2\pi\sigma^2) - \frac{1}{2\sigma^2}\sum_{n=0}^{N-1}(x[n] - E[x[n]])^2$$

$$\frac{\partial \log p(x)}{\partial A} = \frac{2}{\sigma^2}\sum_{n=0}^{N-1}(x[n] - E[x[n]])$$

$$\frac{\partial^2 \log p(x)}{\partial^2 A} = -\frac{2N}{\sigma^2} \cdot \frac{\partial E(x[n])}{\partial A}$$

CRLB $= -\text{ans}^{-1}$

Unbiased estimator with CRLB

$$\Rightarrow \frac{\partial \ln p(x)}{\partial \theta} = I(\theta)[g(x) - \theta]$$

$$\Rightarrow \hat{\theta} = g(x)$$

$\ln\left(\frac{1}{x^y}\right) = y \ln x$ $\ln \prod_{i=1}^{N} z[i] = \sum \ln z[i]$

$\ln e^x = x$

$Var(x)$, $Cov(x, x)$

MVU ~~Estimator~~

$$\hat{\theta}_{MVU} = (H^T \cancel{C^{-1}} H)^{-1} H^T \cancel{C^{-1}} (x - \cancel{x}) = (H^T H)^{-1} H^T x$$

$$C_{\hat{\theta}} = (H^T C^{-1} H)^{-1} = \sigma^2 (H^T H)^{-1}$$

$$Var(\hat{\theta}_{MVU}) = (H^T C^{-1} H)^{-1}$$

BLUE

$x = s\,N + w$

$$\hat{\theta}_{BLUE} = \frac{s^T C^{-1} x}{s^T C^{-1} s} \qquad Var(\hat{\theta}) = \frac{1}{s^T C^{-1} s}$$

MLE

Set $\frac{\partial \log p(x;\theta)}{\partial \theta} = 0$