521348S Statistical Signal Processing I

Second midterm exam, Friday 21.10.2022 08.15 - 10.00

Instructions

Please make sure to return your answers on time. Write your name and your student ID on all papers.

Solve all three (3) problems. Clearly indicate which question you are solving. Intermediate steps need to be included (please do not just write the answer). Simplify your answers as much as possible.

Return the exam answer paper(s) to the invigilator before leaving the exam hall. Show the student card (or mobile student card on the mobile phone) or identity card (passport, ID card, or KELA card with a photo) to the exam invigilator.

Problem 1. Answer **briefly** to the following questions:

(a) What is the main difference between Bayesian and classical approaches to estimation?

(b) Assume posteriori PDF is $p(\theta|x)$. Write equations for MMSE and MAP estimators.

(c) What is the benefit of Vector MAP compared to normal MAP?

(d) What is ROC curve? Plot an example.

(e) What metric is Neyman-Pearson approach to detection maximizing? What is the constraint?

(f) What are false alarms in detection?

Problem 2. Assume signal model

$$s[n] = \begin{cases} A & 0 \leq n \leq M-1 \\ -A & M \leq n \leq N-1 \end{cases}$$

Assume that received noisy samples are $x[n], n = 0,1,\cdots,N-1$. We want to estimate A.

a) Write the least squares (LS) cost $J(A)$ for this problem as an equation. Hint: the answer should be short. You do not have to solve the cost, just write equation for it.

b) Derive the LS estimator for A. Hint: you can use, for example, full derivation or linear least squares using observation matrix $\boldsymbol{H}$. In either case, simplify as much as possible (matrix form for the result is not acceptable).

Problem 3.

a) There are two random variables with joint probability mass function (probability density function) as given by the following table. Find the MMSE estimator for X given Y. Hint: notice that X is here the unknown variable. Hint: remember the Bayes rule.

$$\begin{bmatrix} & Y=0 & Y=1 \\ X=0 & \frac{1}{7} & \frac{3}{7} \\ X=1 & \frac{3}{7} & 0 \end{bmatrix}$$

b) Let us study signal detection problem. Hypothesis H_1 corresponds to DC level A (greater than zero) in additive white Gaussian noise. Hypothesis H_0 corresponds to only white Gaussian noise. There are N samples in both hypothesis. The variance of the noise is σ^2 for both hypothesis. Derive the test using the minimum probability of error criteria for two cases:

(1): The probability of hypothesis H_0 is $P(H_0) = 0.5$. Simplify as much as possible.

(2): The probability of hypothesis H_0 is $P(H_0) = 0.25$. Simplify as much as possible.

For case (1) explain what the threshold means and why it is intuitively correct threshold.

Some equations provided:

$$f(\theta \mid x) = \frac{f(\theta, x)}{f(x)} = \frac{f(\theta)\, f(x \mid \theta)}{f(x)} \qquad \frac{P(H_0)}{P(H_1)}$$

$$\int A p(A \mid \mathbf{x}) dA \qquad L(x) = \frac{p(x; H_1)}{p(x; H_0)} > \gamma$$

$$p(A \mid \mathbf{x}) = \frac{p(\mathbf{x} \mid A)\, p(A)}{p(\mathbf{x})}$$

$$= \frac{p(\mathbf{x} \mid A)\, p(A)}{\int p(\mathbf{x} \mid A)\, p(A) dA} \qquad \left(\mathbf{H}^T \mathbf{H}\right)^{-1} \mathbf{H}^T \mathbf{x}$$