UNIVERSITY OF OULU

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521348S Statistical Signal Processing I

First midterm exam, Thu 26.9.2022 14:15-16:15

Instructions

No devices or material is allowed in the exam.

Please make sure to return your answers on time. Write your name and your student ID on all papers.

Solve all three (3) problems. Clearly indicate which question you are solving. Intermediate steps need to be included (please do not just write the answer). Simplify your answers as much as possible.

Return the exam answer paper(s) to the invigilator before leaving the exam hall. Show the student card (or mobile student card on the mobile phone) or identity card (passport, ID card, or KELA card with a photo) to the exam invigilator.

Problem 1. Answer briefly to the following questions:

(a) What are the requirements for an estimator to be an efficient estimator?

(b) Explain MVU estimators and how CRLB relates to them.

(c) Describe the good asymptotic properties of Maximum Likelihood estimators regarding biasedness, CRLB, and probability density function (PDF).

(d) What information about PDF does BLUE require?

(e) Write the observation matrix $\boldsymbol{H}$ for the linear model for this model: $x[n] = \theta_1 + \theta_2 n + \theta_3 n^2, n = 1,2,3,4$ (here $\theta_1, \theta_2, \theta_3$ are the unknown parameters to be estimated and n is the time index). Write also the $\boldsymbol{\theta}$ vector for the linear model.

(f) Explain the invariance property of Maximum Likelihood estimation.

Problem 2. The data

$$x[n] = 3A + w[n],$$

for $n = 0,1,2,\cdots,N-1$ is observed where $w[n]$ is zero-mean white Gaussian noise with variance σ^2. We want to estimate unknown parameter A based on the observations $x[n]$.

(a) Derive the Cramer Rao lower bound for A.

(b) Find the efficient estimator of A.

Problem 3.

(a) Derive the Maximum Likelihood estimator for parameter θ based on statistically independent measurements $z_1, z_2, \cdots, z_N$ which follow the uniform distribution with range $(\theta-1, \theta+1)$ [the values range from $\theta-1$ to $\theta+1$]. Show all valid solutions.

(b) Find the Cramer-Rao lower bound for estimator of α based on N statistically independent measurements $z_1, z_2, \cdots, z_N$. Individual measurement z_i follows the probability density function

$$p(z_i; \alpha) = \frac{z_i}{\alpha^2}\exp\left(-\frac{z_i^2}{2\alpha^2}\right), z_i \geq 0,$$

where $i = 1,2,\cdots,N$. Hint: $E[z_i^2] = 2\alpha^2$.

Some equations provided (not necessary to use them):

$$\frac{1}{-E\left[\frac{\partial^2 \ln p(\mathbf{x};\theta)}{\partial\theta^2}\right]}$$

$$\frac{\partial \ln p(\mathbf{x};\theta)}{\partial\theta} = I(\theta)(g(\mathbf{x})-\theta)$$

$$\hat{\boldsymbol{\theta}}_{MVU} = \left(\mathbf{H}^T\mathbf{H}\right)^{-1}\mathbf{H}^T\mathbf{x}$$

$$\frac{\sigma^2}{\sum_{n=0}^{N-1}\left[\frac{\partial s[n;\theta]}{\partial\theta}\right]^2}$$

$$\mathbf{C}_{\hat{\boldsymbol{\theta}}} = \sigma^2\left(\mathbf{H}^T\mathbf{H}\right)^{-1}$$