UNIVERSITY OF OULU

YLEISEN TENTIN TENTTILOMAKE - GENERAL EXAM FORM

Opiskelija täyttää / Student fills in

Opiskelijan nimi / Student name:	**Opiskelijanumero / Student number:**

Opettaja täyttää / Lecturer fills in

Opintojakson koodi / The code of the course:
521348S

Opintojakson (tentin) nimi / The name of the course or exam:
Tilastollinen signaalinkäsittely I / Statistical Signal Processing I

Opintopistemäärä / Credit units:
5

Mikäli kyseessä on välikoe, opintopistemääräksi täytetään 0 op.
0 ECTS Credits is used for mid-term exams.

Tiedekunta / Faculty: Tieto- ja sähkötekniikan tiedekunta / Faculty of Information Technology and Electrical Engineering

Tentin pvm / Date of exam:	**Tentin kesto tunteina / Exam in hours:**
2019-11-25	3 h
Tentaattori(t) / Examiner(s):	**Sisäinen postiosoite / Internal address:**
Markku Juntti	9CWC-RT

Tentissä sallitut apuvälineet / The devices allowed in the exam:
Funktiolaskin / Scientific calculator

Muut tenttiä koskevat ohjeet opiskelijalle (esimerkiksi kuinka moneen kysymyksen opiskelijan tulee vastata) / Other instructions for students e.g. how many questions he/she should answer:

To re-take midterm exam #1: Solve problems 1–3. / Ensimmäisen välikokeen uusinta: tehtävät 1-3.

To re-take midterm exam #2: Solve problems 4–6. / Toisen välikokeen uusinta: tehtävät 4-6.

To take the final exam (whole course): Solve freely five (5) problems out of six (6) available. / Loppukoe: valitse vapaasti viisi tehtävää, joihin vastaat.

Mark clearly on the cover paper, which of the exam options mentioned above you take. If you solve all six (6) problems and do not mark otherwise, your grade will be determined as if it were re-take of both mid-term exams.

Merkitse selvästi, miten haluat tenttisuorituksesi arvioitavan.

521348S STATISTICAL SIGNAL PROCESSING I

Final Exam

25 – Nov – 2019

To re-take **midterm exam #1**: Solve **problems 1–3**.
To re-take **midterm exam #2**: Solve **problems 4–6**.
To take the **final exam** (whole course): **Solve five (5) problems** out of six (6) available. You can freely pick any five problems out of six available below.
Mark clearly on the cover paper, which of the exam options mentioned above you take. If you solve all six (6) problems and do not mark otherwise, your grade will be determined as if it were re-take of both mid-term exams.

Mark on the top of each paper to which problem your answer belongs to, if it continues from a different paper. Use of function calculator is allowed, but programmable calculators not.

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1. Answer briefly to the following questions on estimation:

   (a) What are the basic modeling assumptions on the unknwon parameter to be estimated in the classical and Bayesian estimation, respectively?

   (b) What are the definitions of the mean square error (MSE) in classical and Bayesian estimation, respectively?

   (c) What is an unbiased classical estimator? How is the MSE of an unbiased estimator related to its statistical properties?

2. The data

$$x[n] = A\cos(2\pi f_o n + \phi) + w[n]$$

for $n = 0, 1, 2, ...N-1$ are observed, where $w[n]$ is zero-mean WGN with variance σ^2. A and f_o are known with $0 < f_o < 1/2$.

(a) Find the CRLB for estimating the phase ϕ.

(b) Does an efficient estimator exist?

Hint: For $0 < f_o < 1/2$,

$$\sum_{i=0}^{N-1} \cos(4\pi f_o n + 2\phi) \approx 0$$

and

$$\cos^2 x = \frac{1 + \cos 2x}{2}$$

3. A batch of N IID data samples are observed from the following PDFs

(a) Gaussian

$$p(x;\mu) = \frac{1}{\sqrt{2\pi}} e^{[-\frac{1}{2}(x-\mu)^2]}$$

(b) Exponential

$$p(x;\lambda) = \begin{cases} \lambda e^{-(\lambda x)}, & x > 0 \\ 0, & x < 0 \end{cases}$$

In each case, find the ML estimate of the unknown parameters μ and λ, respectively. Be sure to verify that it indeed maximizes the likelihood function. Discuss whether the estimators make sense?

4. Answer briefly to the following questions on detection:

(a) Explain the basic idea of the binary Neyman-Pearson detection and the figures of merit it optimizes.

(b) What is the receiver operating characteristics? Explain the trade-off it describes.

(c) Explain the basic idea of the binary Bayesian detection and the figure of merit it optimizes.

5. Let us consider the signal model

$$s[n] = \begin{cases} \theta, & 0 \leqslant n \leqslant K-1 \\ -3\theta, & K \leqslant n \leqslant L-1 \end{cases}$$

(a) Find the LS estimate of θ and the corresponding minimum squared error.

(b) Assume further that data $x[n] = s[n] + w[n]$ for $n = 0, 1, 2, \ldots L-1$ are observed. If now $w[n]$ is zero-mean WGN with variance σ^2, find the PDF of the LS estimate.

6. We want to estimate random variable θ. We observe noisy data $x[n]$ for $n = 0, 1, 2, \ldots N-1$. The observed samples are the independent and have the conditional PDF,

$$p(x[n]|\theta) = \begin{cases} e^{-(x[n]-\theta)}, & x[n] > \theta \\ 0, & \text{otherwise} \end{cases}$$

The prior PDF is

$$p(\theta) = \begin{cases} \lambda e^{-\lambda\theta}, & \theta > 0 \\ 0, & \text{otherwise} \end{cases}$$

with $\text{var}[\theta] = \frac{1}{\lambda^2}$

(a) Determine the MAP estimator for θ assuming that $\text{var}[\theta]$ approaches infinity. Hint: remember what happens to MAP when prior is uninformative.

(b) Determine the MAP estimator for θ for general case (arbitrary values of $\text{var}[\theta]$).

(c) Find the pdf $p(x)$ for the received vector $\mathbf{x}$ (containing all the observed samples).