Signaalianalyysi, 1.välikoe 14.12.2013, lyhennetyt ratkaisut

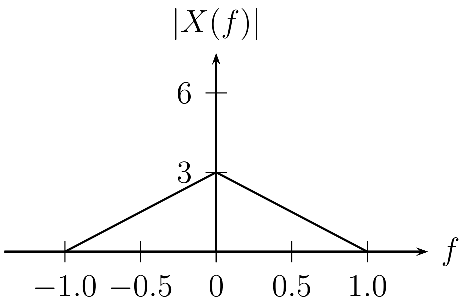
1. (a) $E_x = \sum_{-\infty}^{\infty} |x(n)|^2 = \sum_0^3 1 = 4 < \infty \;\Rightarrow\; x(n)$ on energiasignaali

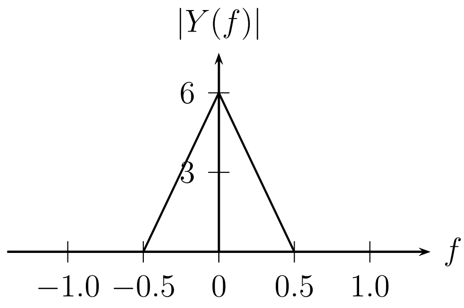
   (b) $r_{xy}(m) = x(-n) * y(n) = \{6, -6, -1\underset{\uparrow}{9}, 4, 10\}$, $x(n) \circledast y(n) = \{-1\underset{\uparrow}{0}, -11, 16\}$

   (c) $y(n) = x(n) * h(n) = \{-2, \underset{\uparrow}{0}, 5, 0, -3\}$

2. (a) $X(f) = 3\,\mathrm{tri}(f)e^{-j4\pi f}$, $|X(f)| = 3\,\mathrm{tri}(f)$

   $Y(f) = 2X(2f)$, $|Y(f)| = 6\,\mathrm{tri}(2f)$



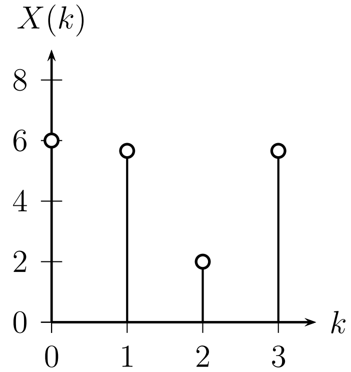


   (b) Diskreetti signaali $x(n) = x(t)\big|_{t=nT=\frac{n}{3000}} = 2\cos(\frac{\pi}{2}n) - 3\sin(\frac{2\pi}{3}n - \frac{\pi}{5})$.

   Digitaaliset taajuudet $\omega_1 = \frac{\pi}{2}$ ja $\omega_2 = \frac{2\pi}{3}$.

3. (a) $X(k) = \sum_{n=0}^{3} x(n)e^{-j\frac{\pi}{2}kn},\; k = 0, 1, 2, 3 \Rightarrow X(k) = \{\underset{\uparrow}{6}, 4+4j, 2, 4-4j\}$

   Amplitudispektri $|X(k)| = \{\underset{\uparrow}{6}, 4\sqrt{2}, 2, 4\sqrt{2}\}$



   (b) $X(\omega) = \sum_{n=-\infty}^{\infty} x(n)e^{-j\omega n} = \frac{1}{3}[e^{j\omega} + 1 + e^{-j\omega}] = \frac{1}{3}[1 + 2\cos(\omega)],\; -\pi < \omega \leq \pi$

   Amplitudispektri $|X(\omega)| = \frac{1}{3}|1 + 2\cos(\omega)|,\; -\pi < \omega \leq \pi$.

4. (a) Fourier-muunnettu differentiaaliyhtälö: $(j2\pi f)^2 Y(f) - \frac{1}{4}Y(f) = -X(f)e^{-j2\pi f}$.

   Siirtofunktio $H(f) = \frac{Y(f)}{X(f)} = \frac{e^{-j2\pi f}}{\frac{1}{4} + (2\pi f)^2}$.

   Impulssivaste $h(t) = \mathcal{F}^{-1}\{H(f)\} = e^{-\frac{1}{2}|t-1|}$.

   Vaihevaste $\theta(f) = \arg[H(f)] = -2\pi f$.

   (b) Z-muunnettu diff.yht. $Y(z) = -\frac{9}{4}Y(z)z^{-2} + X(z) + \frac{1}{3}X(z)z^{-2}$.

   Siirtofunktio $H(z) = \frac{Y(z)}{X(z)} = \frac{1 + \frac{1}{3}z^{-2}}{1 + \frac{9}{4}z^{-2}}$.

   Navat $z = \pm\frac{3}{2}j$ eivät ole yksikköympyrän sisällä $\Rightarrow$ ei ole stabiili.