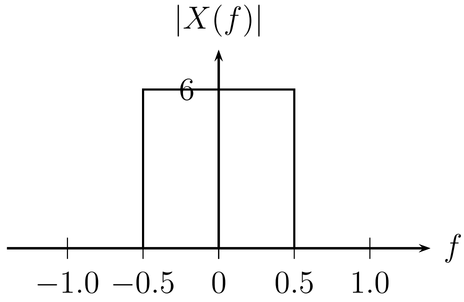
Signaalianalyysi, 1.välikoe 15.12.2012, lyhennetyt ratkaisut

1. (a) $E_x = \int_{-\infty}^{\infty} |\mathrm{tri}(t)|^2 dt = 2\int_0^1 (1-t)^2 dt = 2\Big/_0^1 -\frac{(1-t)^3}{3} = \frac{2}{3} < \infty \;\Rightarrow\; x(t)$ on energiasignaali

   (b) $P_x = \lim_{M\to\infty} \frac{1}{2M+1} \sum_{n=-M}^{M} |u(n)|^2 = \lim_{M\to\infty} \frac{1}{2M+1} \sum_{n=0}^{M} 1$
   $= \lim_{M\to\infty} \frac{M+1}{2M+1} = \frac{1}{2} < \infty \;\Rightarrow\; x(n)$ on tehosignaali

   (c) $r_{xx}(n) = x(-n) * x(n) = \{-8, 2, -4, \underset{\uparrow}{21}, -4, 2, -8\}$

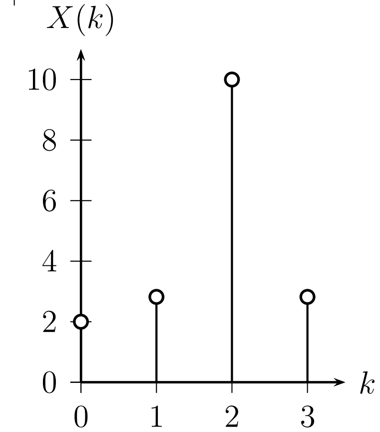
2. (a) $X(f) = 6\,\mathrm{rect}(f)e^{-j2\pi f}$, $|X(f)| = 6\,\mathrm{rect}(f)$



   (b) $X(\omega) = \sum_{n=-\infty}^{\infty} x(n)e^{-j\omega n} = \frac{1}{2} + 2e^{-j\omega} - 2e^{-j2\omega} - \frac{1}{2}e^{-j3\omega}$
   $= e^{j(\frac{\pi}{2} - \frac{3\omega}{2})}[\sin(\frac{3\omega}{2}) + 4\sin(\frac{\omega}{2})],\; -\pi < \omega \le \pi$
   Amplitudispektri $|X(\omega)| = |\sin(\frac{3\omega}{2}) + 4\sin(\frac{\omega}{2})|,\; -\pi < \omega \le \pi$
   Vaihespektri $\theta(\omega) = \begin{cases} \frac{\pi}{2} - \frac{3\omega}{2}, \sin(\frac{3\omega}{2}) + 4\sin(\frac{\omega}{2}) > 0 \\ \frac{3\pi}{2} - \frac{3\omega}{2}, \sin(\frac{3\omega}{2}) + 4\sin(\frac{\omega}{2}) < 0 \end{cases}$.

3. (a) Näytteenottoaika $T_0 = NT = 400 \cdot 0.01\,s = 4\,s$, spektrin resoluutio $\Delta f = \frac{1}{T_0} = \frac{1}{4}Hz$.
   Analogiset taajuudet $\{f_1, f_2, f_3\} = \frac{1}{4} \cdot \{20, 100, 180\}Hz = \{5,\ 25,\ 45\}Hz$.

   (b) $X(k) = \sum_{n=0}^{3} x(n)e^{-j\frac{\pi}{2}kn},\; k = 0,1,2,3 \Rightarrow\; X(k) = \{-\underset{\uparrow}{2}, -2-2j, 10, -2+2j\}$
   Amplitudispektri $|X(k)| = \{\underset{\uparrow}{2}, 2\sqrt{2}, 10, 2\sqrt{2}\}$



4. (a) Fourier-muunnettu differentiaaliyhtälö: $j2\pi f Y(f) + 5Y(f) = 4X(f)e^{-j6\pi f}$.
   Siirtofunktio $H(f) = \frac{Y(f)}{X(f)} = \frac{4e^{-j6\pi f}}{5+j2\pi f}$.
   Impulssivaste $h(t) = \mathcal{F}^{-1}\{H(f)\} = 4e^{-5(t-3)}u(t-3)$.
   Systeemi on kausaalinen, koska $h(t) = 0$ kun $t < 0$.

   (b) Z-muunnettu diff.yht. $Y(z) = Y(z)z^{-1} - \frac{3}{16}Y(z)z^{-2} + X(z) + \frac{1}{2}X(z)z^{-1}$.
   Siirtofunktio $H(z) = \frac{Y(z)}{X(z)} = \frac{1+\frac{1}{2}z^{-1}}{1-z^{-1}+\frac{3}{16}z^{-2}}$.
   Navat $z = \frac{1}{4}$ ja $z = \frac{3}{4}$ ovat yksikköympyrän sisällä $\Rightarrow$ on stabiili.