Introduction to Optimization

Some tasks from the old exams

1. Using the Newton method find the approximation to the minimum of the unconstrained optimization probem

$$\min_{x\in\mathbb{R}^2} x_1^2 + x_2^2 + \mathrm{e}^{x_1} - x_2.$$

Use $x^0 = [0\ 0]^\top$ as the starting point for your iterations. Compute two iterations.

2. Solve the quadratic optimization problem

$$\min_{x\in\mathbb{R}^2} 2x_1^2 - 2x_1x_2 + 4x_2^2 - x_1 - 2x_2$$

using the conjugate gradient method with the initial guess $x^0 = [0\ 0]^\top$.

3. Solve the quadratic optimization problem

$$\min_{x\in\mathbb{R}^2} 2x_1^2 + x_1x_2 + x_2^2 - 4x_1 + x_2$$

using the conjugate gradient method with the initial guess $x^0 = [0\ 0]^\top$.

4. Find all the points that satisfy the Karush-Kuhn-Tucker conditions for the constrained optimization problem

$$\begin{aligned} \min \quad & x_1^2 + x_1x_2 + 2x_2^2 - 2x_1 \\ x_1 - x_2 - 2 \leq 0 & \\ x_1 \leq 2. & \end{aligned}$$

5. Find all the points that satisfy the Karush-Kuhn-Tucker conditions for the constrained optimization problem

$$\begin{aligned} \min \quad & x_1 - 2x_2 \\ x_1 - x_2^2 + 1 \geq 0 & \\ x_2 \geq 0. & \end{aligned}$$

6. Find the dual function and the dual problem of the constrained optimization problem:

$$\min_{2x_1+x_2\leq -2} 2x_1^2 + x_2^2 - x_1x_2 - x_2.$$

7. Find the dual function and solve the dual problem of the constrained optimization problem:

$$\min_{2x_1+x_2\leq -1} 2x_1^2 + x_2^2 - x_1x_2 - x_1.$$

8. Using the Uzawa's algorithm find the approximation to the minimum of the constrained optimization probem

$$\min_{-x_1+2x_2-2\leq 0} x_1^2 + x_2^2 + x_1x_2 - 3x_2.$$

Compute two iterations. Take step size $\rho = \frac{1}{7}$.