

031051S Numerical Matrix Analysis, the 1st partial exam.

Enough details, please.

1. Let $A = \begin{bmatrix} 2 & 2 \\ 1 & 1 \end{bmatrix}$.
 - a) What is the operator norm of A ?
 - b) What is the Frobenius norm of A ?
 - c) Give the best rank-1 approximation to A .
2. Let $\mathcal{V} \subset \mathbb{C}^{2 \times 2}$ be the set of Toeplitz matrices. Show that \mathcal{V} is an invertible matrix subspace of $\mathbb{C}^{2 \times 2}$ by finding \mathcal{V}^{-1} . (Hint: find the inverse of $\begin{bmatrix} t_0 & t_1 \\ t_2 & t_0 \end{bmatrix}$.)
3. Let $A = \begin{bmatrix} 2 & 2 & 1 \\ 2 & 2 & -1 \\ 1 & 0 & 1 \end{bmatrix}$. Compute the partially pivoted LU factorization of A . Is A invertible? (Give careful arguments.)
4. Let $A = \begin{bmatrix} 4 & 4 \\ 4 & 5 \end{bmatrix}$. Name at least three factorizations to solve the linear system $Ax = b$. Compute the factorization you should use in practice for problems of this type.