031075P MATEMATIIKAN PERUSKURSSI II

1. välikoe 23.01.2020

VÄLIVAIHEET JA PERUSTELUT NÄKYVIIN, KIITOS!

1. a) Tutki vertailuperiaatetta käyttäen sarjan

$$\sum_{k=1}^{\infty} \frac{3}{5\sqrt{k}-2}$$

suppenemista. (2p)

b) Tutki integraalitestiä käyttäen sarjan

$$\sum_{k=1}^{\infty} k\,\mathrm{e}^{-k}$$

suppenemista. Osoita tarkasti perustellen, että integraalitestin oletukset ovat voimassa. (3p)
Käytä osittaisintegroinnin kaavaa

$$\int_a^b u(x)v'(x)\,\mathrm{d}x = \Big/_{\!a}^{\,b}\, u(x)v(x) - \int_a^b u'(x)v(x)\,\mathrm{d}x.$$

c) Potenssisarjan

$$\sum_{k=0}^{\infty} \frac{6^k}{k+1}\left(x-\frac{1}{6}\right)^k$$

suppenemisväli on $0 < x < \frac{1}{3}$. Määrää potenssisarjan suppenemissäde R. (1p)

d) Funktion f Taylorin polynomi pisteessä x_0 lasketaan kaavalla

$$T_n(x) = \sum_{k=0}^{n} \frac{f^{(k)}(x_0)}{k!}(x-x_0)^k.$$

Laske funktiolle $f(x) = \sqrt[3]{1+x}$ Taylorin polynomi $T_1(x)$ kehityskeskuksena $x_0 = 0$. Arvioi funktion arvoa $f(0.331)$ saadun Taylorin polynomin avulla. (2p)

KAAVAKOKOELMA LIITTEENÄ

$$\mathrm{e}^x = \sum_{k=0}^{\infty} \frac{x^k}{k!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots, \ x \in \mathbb{R}$$

$$\cos x = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k}}{(2k)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots, \ x \in \mathbb{R}$$

$$\sin x = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{(2k+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots, \ x \in \mathbb{R}$$

$$\cosh x = \sum_{k=0}^{\infty} \frac{x^{2k}}{(2k)!} = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \cdots, \ x \in \mathbb{R}$$

$$\sinh x = \sum_{k=0}^{\infty} \frac{x^{2k+1}}{(2k+1)!} = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + \cdots, \ x \in \mathbb{R}$$

$$\frac{1}{1-x} = \sum_{k=0}^{\infty} x^k = 1 + x + x^2 + x^3 + \cdots, \ |x| < 1$$

$$\ln(1+x) = \sum_{k=0}^{\infty} \frac{(-1)^k x^{k+1}}{k+1} = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots, \ |x| < 1$$

$$\overline{\mathrm{arc}} \tan x = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{2k+1} = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots, \ |x| < 1$$

$$S(x) = \frac{a_0}{2} + \sum_{k=1}^{\infty} [a_k \cos(kx) + b_k \sin(kx)]$$

$$a_0 = \frac{1}{\pi} \int_0^{2\pi} f(t) \ \mathrm{d}t \quad a_k = \frac{1}{\pi} \int_0^{2\pi} f(t) \cos(kt) \ \mathrm{d}t \quad b_k = \frac{1}{\pi} \int_0^{2\pi} f(t) \sin(kt) \ \mathrm{d}t$$

$$D(x,y) = f_{xx}(x,y) f_{yy}(x,y) - [f_{xy}(x,y)]^2$$

$$\begin{cases} x &= \rho \sin\theta \cos\varphi \\ y &= \rho \sin\theta \sin\varphi \\ z &= \rho \cos\theta \end{cases} \quad \left| \frac{\partial(x,y,z)}{\partial(\rho,\theta,\varphi)} \right| = \rho^2 \sin\theta \quad \oint_{\partial A} P \ \mathrm{d}x + Q \ \mathrm{d}y = \iint_A \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) \mathrm{d}A$$

$$\iint_S F(x,y,z) \, \mathrm{d}S = \iint_A F(x,y,f(x,y)) \sqrt{1 + [f_x(x,y)]^2 + [f_y(x,y)]^2} \, \mathrm{d}A$$

$$\oint_{\partial S} \vec{F} \cdot \mathrm{d}\vec{x} = \iint_S \nabla \times \vec{F} \cdot \vec{n}^0 \, \mathrm{d}S \quad \iint_S \vec{F} \cdot \vec{n}^0 \, \mathrm{d}S = \iiint_V \nabla \cdot \vec{F} \, \mathrm{d}V$$

$$D(\frac{f(x)}{g(x)}) = \frac{f'(x)g(x) - g'(x)f(x)}{[g(x)]^2} \quad D(f(x)g(x)) = f'(x)g(x) + f(x)g'(x) \quad Dx^n = nx^{n-1}$$

$$D([f(x)]^n) = n[f(x)]^{n-1} f'(x) \quad D\,\mathrm{e}^{f(x)} = \mathrm{e}^{f(x)} f'(x) \quad D \ln|f(x)| = \frac{f'(x)}{f(x)}$$

$$D\overline{\mathrm{arc}} \tan x = \frac{1}{1+x^2} \quad D \sin x = \cos x \quad D \cos x = -\sin x$$

$$\int x^n \ \mathrm{d}x = \frac{x^{n+1}}{n+1} + C \ (n \neq -1) \quad \int \frac{1}{x} \ \mathrm{d}x = \ln|x| + C$$

$$\int f'(x)[f(x)]^n \ \mathrm{d}x = \frac{[f(x)]^{n+1}}{n+1} + C \ (n \neq -1) \quad \int \frac{f'(x)}{f(x)} \ \mathrm{d}x = \ln|f(x)| + C$$

$$\int f'(x) \, \mathrm{e}^{f(x)} \ \mathrm{d}x = \mathrm{e}^{f(x)} + C \quad \int \frac{\mathrm{d}x}{1+x^2} = \overline{\mathrm{arc}} \tan x + C$$

$$\int \sin x \ \mathrm{d}x = -\cos x + C \quad \int \cos x \ \mathrm{d}x = \sin x + C$$