

## 031075P MATEMATIIKAN PERUSKURSSI II

1. välikoe 26.1.2016

VÄLIVAIHEET JA PERUSTELUT NÄKYVIIN, KIITOS!

1. a) Osoita sarja

$$\sum_{k=1}^{\infty} (\sqrt{k-1} - \sqrt{k})$$

hajaantuvaksi osasummien jonon

$$(S_n)_{n=1}^{\infty}, S_n = \sum_{k=1}^n (\sqrt{k-1} - \sqrt{k}), n = 1, 2, 3, \dots,$$

avulla. (2p)

b) Osoita sarja  $\sum_{k=1}^{\infty} k e^{-k^2}$  suppenevaksi integraalitestin avulla. (2p)

c) Laske potenssisarjojen avulla raja-arvo

$$\lim_{x \rightarrow 0} \frac{\sinh(3x) - 3x \cosh(x)}{x^2 \ln(1+x)}. \quad (4p)$$

KAAVAKOKOELMA LIITTEENÄ

$$e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots, \quad x \in \mathbb{R}$$

$$\cos x = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k}}{(2k)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots, \quad x \in \mathbb{R}$$

$$\sin x = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{(2k+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots, \quad x \in \mathbb{R}$$

$$\cosh x = \sum_{k=0}^{\infty} \frac{x^{2k}}{(2k)!} = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \cdots, \quad x \in \mathbb{R}$$

$$\sinh x = \sum_{k=0}^{\infty} \frac{x^{2k+1}}{(2k+1)!} = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + \cdots, \quad x \in \mathbb{R}$$

$$\frac{1}{1-x} = \sum_{k=0}^{\infty} x^k = 1 + x + x^2 + x^3 + \cdots, \quad |x| < 1$$

$$\ln(1+x) = \sum_{k=0}^{\infty} \frac{(-1)^k x^{k+1}}{k+1} = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots, \quad |x| < 1$$

$$\overline{\arctan} x = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{2k+1} = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots, \quad |x| < 1$$

$$S(x) = \frac{a_0}{2} + \sum_{k=1}^{\infty} [a_k \cos(kx) + b_k \sin(kx)]$$

$$a_0 = \frac{1}{\pi} \int_0^{2\pi} f(t) dt \quad a_k = \frac{1}{\pi} \int_0^{2\pi} f(t) \cos(kt) dt \quad b_k = \frac{1}{\pi} \int_0^{2\pi} f(t) \sin(kt) dt$$

$$D(x, y) = f_{xx}(x, y)f_{yy}(x, y) - [f_{xy}(x, y)]^2$$

$$\begin{cases} x = \rho \sin \theta \cos \varphi \\ y = \rho \sin \theta \sin \varphi \\ z = \rho \cos \theta \end{cases} \quad \left| \frac{\partial(x, y, z)}{\partial(\rho, \theta, \varphi)} \right| = \rho^2 \sin \theta \quad \oint_{\partial A} P dx + Q dy = \iint_A \left( \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dA$$

$$\iint_S F(x, y, z) dS = \iint_A F(x, y, f(x, y)) \sqrt{1 + [f_x(x, y)]^2 + [f_y(x, y)]^2} dA$$

$$\oint_{\partial S} \vec{F} \cdot d\vec{x} = \iint_S \nabla \times \vec{F} \cdot \vec{n}^0 dS \quad \iint_S \vec{F} \cdot \vec{n}^0 dS = \iiint_V \nabla \cdot \vec{F} dV$$

$$D\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x)g(x) - g'(x)f(x)}{[g(x)]^2} \quad D(f(x)g(x)) = f'(x)g(x) + f(x)g'(x) \quad Dx^n = nx^{n-1}$$

$$D([f(x)]^n) = n[f(x)]^{n-1}f'(x) \quad De^{f(x)} = e^{f(x)}f'(x) \quad D \ln |f(x)| = \frac{f'(x)}{f(x)}$$

$$D \overline{\arctan} x = \frac{1}{1+x^2} \quad D \sin x = \cos x \quad D \cos x = -\sin x$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C \quad (n \neq -1) \quad \int \frac{1}{x} dx = \ln |x| + C$$

$$\int f'(x)[f(x)]^n dx = \frac{[f(x)]^{n+1}}{n+1} + C \quad (n \neq -1) \quad \int \frac{f'(x)}{f(x)} dx = \ln |f(x)| + C$$

$$\int f'(x)e^{f(x)} dx = e^{f(x)} + C \quad \int \frac{dx}{1+x^2} = \overline{\arctan} x + C$$

$$\int \sin x dx = -\cos x + C \quad \int \cos x dx = \sin x + C$$