TIETO- JA SÄHKÖTEKNIIKAN TIEDEKUNTA, MATEMATIIKAN JAOS

MATEMATIIKAN PERUSKURSSI II

1. välikoe 30.1.2014

VÄLIVAIHEET JA PERUSTELUT NÄKYVIIN, KIITOS!

1. a) Osoita sarja $\sum\limits_{k=1}^{\infty} \dfrac{1}{k^4 + 9}$ suppenevaksi vertailuperiaatteen avulla. (2p)

   b) Tutki sarjan $\sum\limits_{k=1}^{\infty} \sqrt{k + 1}$ suppenemista. (2p)

   c) Laske integraalitestin avulla suppenevan sarjan

   $$\sum_{k=1}^{\infty} (k + 1)e^{-k^2 - 2k}$$

   summalle likiarvo, jonka virhe on pienempi kuin $2 \cdot 10^{-7}$. Virhearvioon liittyvät oletukset ovat voimassa. (3p)

   d) Tutki perustellen, suppeneeko potenssisarja

   $$\sum_{k=1}^{\infty} \frac{3}{k^2}(x - 2)^k$$

   pisteessä $x = 1$. (1p)

KAAVAKOKOELMA KÄÄNTÖPUOLELLA

$$e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots, \quad x \in \mathbb{R}$$

$$\cos x = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k}}{(2k)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots, \quad x \in \mathbb{R}$$

$$\sin x = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{(2k+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots, \quad x \in \mathbb{R}$$

$$\cosh x = \sum_{k=0}^{\infty} \frac{x^{2k}}{(2k)!} = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \cdots, \quad x \in \mathbb{R}$$

$$\sinh x = \sum_{k=0}^{\infty} \frac{x^{2k+1}}{(2k+1)!} = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + \cdots, \quad x \in \mathbb{R}$$

$$\frac{1}{1-x} = \sum_{k=0}^{\infty} x^k = 1 + x + x^2 + x^3 + \cdots, \quad |x| < 1$$

$$\ln(1+x) = \sum_{k=0}^{\infty} \frac{(-1)^k x^{k+1}}{k+1} = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots, \quad |x| < 1$$

$$\overline{\arc} \tan x = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{2k+1} = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots, \quad |x| < 1$$

$$S(x) = \frac{a_0}{2} + \sum_{k=1}^{\infty} [a_k \cos(kx) + b_k \sin(kx)]$$

$$a_0 = \frac{1}{\pi} \int_0^{2\pi} f(t)\, dt \quad a_k = \frac{1}{\pi} \int_0^{2\pi} f(t) \cos(kt)\, dt \quad b_k = \frac{1}{\pi} \int_0^{2\pi} f(t) \sin(kt)\, dt$$

$$D(x,y) = f_{xx}(x,y) f_{yy}(x,y) - [f_{xy}(x,y)]^2$$

$$\begin{cases} x = \rho \sin\theta \cos\varphi \\ y = \rho \sin\theta \sin\varphi \\ z = \rho \cos\theta \end{cases} \quad \left| \frac{\partial(x,y,z)}{\partial(\rho,\theta,\varphi)} \right| = \rho^2 \sin\theta \quad \oint_{\partial A} P\, dx + Q\, dy = \iint_A \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dA$$

$$\iint_S F(x,y,z)\, dS = \iint_A F(x,y,f(x,y)) \sqrt{1 + [f_x(x,y)]^2 + [f_y(x,y)]^2}\, dA$$

$$\oint_{\partial S} \vec{F} \cdot d\vec{x} = \iint_S \nabla \times \vec{F} \cdot \vec{n}^0\, dS \quad \iint_S \vec{F} \cdot \vec{n}^0\, dS = \iiint_V \nabla \cdot \vec{F}\, dV$$

$$D\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x)g(x) - g'(x)f(x)}{[g(x)]^2} \quad D(f(x)g(x)) = f'(x)g(x) + f(x)g'(x) \quad Dx^n = nx^{n-1}$$

$$D([f(x)]^n) = n[f(x)]^{n-1} f'(x) \quad De^{f(x)} = e^{f(x)} f'(x) \quad D\ln|f(x)| = \frac{f'(x)}{f(x)}$$

$$D\,\overline{\arc}\tan x = \frac{1}{1+x^2} \quad D\sin x = \cos x \quad D\cos x = -\sin x$$

$$\int x^n\, dx = \frac{x^{n+1}}{n+1} + C \ (n \neq -1) \quad \int \frac{1}{x}\, dx = \ln|x| + C$$

$$\int f'(x)[f(x)]^n\, dx = \frac{[f(x)]^{n+1}}{n+1} + C \ (n \neq -1) \quad \int \frac{f'(x)}{f(x)}\, dx = \ln|f(x)| + C$$

$$\int f'(x)e^{f(x)}\, dx = e^{f(x)} + C \quad \int \frac{dx}{1+x^2} = \overline{\arc}\tan x + C$$

$$\int \sin x\, dx = -\cos x + C \quad \int \cos x\, dx = \sin x + C$$