031010P MATEMATIIKAN PERUSKURSSI I

3. välikoe 26.10.2017

VÄLIVAIHEET JA PERUSTELUT NÄKYVIIN, KIITOS!

1. a) Osoita tarkasti derivaatan avulla, että funktio $f(x) = \ln(2x^2 + 3) - 2x^2 - 3$ on aidosti vähenevä, kun $x > 0$. (2p)

   b) Laske L'Hospitalin säännöllä raja-arvo

$$\lim_{x \to 0} \frac{x\cos(x) - \sin(x)}{\cos(3x) - 1}. \quad (4p)$$

   c) Määrää ne käyrän

$$\begin{cases} x = 2t - 3 \\ y = 8\overline{\text{arc}}\tan(t) + 1, t \in \mathbb{R}, \end{cases}$$

   pisteet, joissa tangentin kulmakerroin on 4. (2p)

2. a) Laske

$$\int_0^1 \frac{9x}{(x+1)(x-2)^2}\, dx. \quad (4p)$$

   b) Käyrä

$$y = f(x) = \sqrt{x\cos(x)}, 0 \le x \le \frac{\pi}{2},$$

   pyörähtää x-akselin ympäri. Laske muodostuneen kappaleen tilavuus osittaisintegroimalla. (4p)

KAAVAKOKOELMA LIITTEENÄ

$$\vec{u}\cdot\vec{v}=\sum_{i=1}^{3}u_iv_i \qquad \vec{u}\times\vec{v}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ u_1 & u_2 & u_3\\ v_1 & v_2 & v_3\end{vmatrix}$$

$$d(P_1,l)=\frac{\|\vec{u}\times(\overrightarrow{AP_1})\|}{\|\vec{u}\|} \qquad d(P_1,T)=\frac{|\vec{n}\cdot(\overrightarrow{AP_1})|}{\|\vec{n}\|}$$

x	$\cos x$	$\sin x$
$\frac{\pi}{6}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$
$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$
$\frac{\pi}{3}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$

$$\sin^2 x+\cos^2 x=1 \qquad \tan x=\frac{\sin x}{\cos x} \qquad \cot x=\frac{1}{\tan x}$$

$$\sin x=-\sin(-x)=\cos\left(\frac{\pi}{2}-x\right) \qquad \cos x=\cos(-x)=\sin\left(\frac{\pi}{2}-x\right)$$

$$\sin 2x=2\sin x\cos x \qquad \cos 2x=\cos^2 x-\sin^2 x=2\cos^2 x-1=1-2\sin^2 x$$

$$\frac{a}{\sin\alpha}=\frac{b}{\sin\beta}=\frac{c}{\sin\gamma} \qquad a^2=b^2+c^2-2bc\cos\alpha$$

$$\sin(x+y)=\sin x\cos y+\cos x\sin y \qquad \cos(x+y)=\cos x\cos y-\sin x\sin y$$

$$\sinh x=\frac{1}{2}(e^x-e^{-x}) \qquad \cosh x=\frac{1}{2}(e^x+e^{-x}) \qquad \tanh x=\frac{\sinh x}{\cosh x} \qquad \coth x=\frac{\cosh x}{\sinh x}$$

$$\cosh^2 x-\sinh^2 x=1$$

$$Dx^n=nx^{n-1} \qquad D\sin x=\cos x \qquad D\cos x=-\sin x \qquad D\tan x=\frac{1}{\cos^2 x}=1+\tan^2 x$$

$$De^x=e^x \qquad Da^x=a^x\ln a(a>0) \qquad D\ln|x|=\frac{1}{x} \qquad D\log_a|x|=\frac{1}{x\ln a}(a>0,a\neq 1)$$

$$D\overline{\text{arc}}\sin x=\frac{1}{\sqrt{1-x^2}} \qquad D\overline{\text{arc}}\cos x=-\frac{1}{\sqrt{1-x^2}} \qquad D\overline{\text{arc}}\tan x=\frac{1}{1+x^2}$$

$$(f^{-1})'(y_0)=\frac{1}{f'(x_0)},\ \text{missä } y_0=f(x_0)$$

$$\int x^n\,dx=\frac{x^{n+1}}{n+1}+C\ (n\neq -1) \qquad \int\frac{1}{x}\,dx=\ln|x|+C \qquad \int\tan x\,dx=-\ln|\cos x|+C$$

$$\int\frac{dx}{\cos^2 x}=\int(1+\tan^2 x)\,dx=\tan x+C \qquad \int\frac{dx}{\sin^2 x}=\int(1+\cot^2 x)\,dx=-\cot x+C$$

$$\int\frac{dx}{\sqrt{1-x^2}}=\overline{\text{arc}}\ \sin x+C \qquad \int\frac{dx}{1+x^2}=\overline{\text{arc}}\ \tan x+C$$

$$A=\int_a^b|f(x)|\,dx \qquad V=\pi\int_a^b(f(x))^2\,dx$$

$$\begin{aligned}
Q(x) &= (x-x_1)^{k_1}\dots(x-x_r)^{k_r}(x^2+c_1x+d_1)^{l_1}\dots(x^2+c_sx+d_s)^{l_s};\\
\frac{P(x)}{Q(x)} &= \frac{A_{1,1}}{x-x_1}+\dots+\frac{A_{1,k_1}}{(x-x_1)^{k_1}}+\dots+\frac{A_{r,1}}{x-x_r}+\dots+\frac{A_{r,k_r}}{(x-x_r)^{k_r}}\\
&+\frac{B_{1,1}x+C_{1,1}}{x^2+c_1x+d_1}+\dots+\frac{B_{1,l_1}x+C_{1,l_1}}{(x^2+c_1x+d_1)^{l_1}}+\dots\\
&+\frac{B_{s,1}x+C_{s,1}}{x^2+c_sx+d_s}+\dots+\frac{B_{s,l_s}x+C_{s,l_s}}{(x^2+c_sx+d_s)^{l_s}}
\end{aligned}$$