

031010P MATEMATIIKAN PERUSKURSSI I

3. välikoe 20.10.2015

VÄLIVAIHEET JA PERUSTELUT NÄKYVIIN, KIITOS!

1. a) Laske raja-arvo

$$\lim_{x \rightarrow 0} \frac{\sin(3x) - 3x}{2 \cos(x) + x^3 - 2}. \quad (3\text{p})$$

b) Määrittää funktion $f(x) = x^2 e^x$ kriittiset pisteet sekä tutki niiden laatu toisen derivaatan $f''(x)$ avulla. (5p)

2. a) Laske sijoitusta $t = \sqrt{x}$ käyttäen

$$\int \frac{3}{\sqrt{x}(\sqrt{x} + 1)^3} dx. \quad (4\text{p})$$

b) Käyrä

$$y = f(x) = \sqrt{\frac{2x + 3}{x^2 + 3x + 3}}, \quad 0 \leq x \leq 1,$$

pyörähtää x -akselin ympäri. Laske muodostuneen kappaleen tilavuus. (4p)

KAAVAKOKOELMA TOISELLA PAPERILLA

$$\vec{u} \cdot \vec{v} = \sum_{i=1}^3 u_i v_i \quad \vec{u} \times \vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$$

$$d(P_1, l) = \frac{\|\vec{u} \times (\overrightarrow{AP_1})\|}{\|\vec{u}\|} \quad d(P_1, T) = \frac{|\vec{n} \cdot (\overrightarrow{AP_1})|}{\|\vec{n}\|}$$

x	$\cos x$	$\sin x$
$\frac{\pi}{6}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$
$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$
$\frac{\pi}{3}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$

$$\sin^2 x + \cos^2 x = 1 \quad \tan x = \frac{\sin x}{\cos x} \quad \cot x = \frac{1}{\tan x}$$

$$\sin x = -\sin(-x) = \cos\left(\frac{\pi}{2} - x\right) \quad \cos x = \cos(-x) = \sin\left(\frac{\pi}{2} - x\right)$$

$$\sin 2x = 2 \sin x \cos x \quad \cos 2x = \cos^2 x - \sin^2 x = 2 \cos^2 x - 1 = 1 - 2 \sin^2 x$$

$$\frac{a}{\sin \alpha} = \frac{b}{\sin \beta} = \frac{c}{\sin \gamma} \quad a^2 = b^2 + c^2 - 2bc \cos \alpha$$

$$\sin(x+y) = \sin x \cos y + \cos x \sin y \quad \cos(x+y) = \cos x \cos y - \sin x \sin y$$

$$\sinh x = \frac{1}{2}(e^x - e^{-x}) \quad \cosh x = \frac{1}{2}(e^x + e^{-x}) \quad \tanh x = \frac{\sinh x}{\cosh x} \quad \coth x = \frac{\cosh x}{\sinh x}$$

$$\cosh^2 x - \sinh^2 x = 1$$

$$Dx^n = nx^{n-1} \quad D \sin x = \cos x \quad D \cos x = -\sin x \quad D \tan x = \frac{1}{\cos^2 x} = 1 + \tan^2 x$$

$$De^x = e^x \quad Da^x = a^x \ln a (a > 0) \quad D \ln |x| = \frac{1}{x} \quad D \log_a |x| = \frac{1}{x \ln a} (a > 0, a \neq 1)$$

$$D \overline{\arcsin} x = \frac{1}{\sqrt{1-x^2}} \quad D \overline{\arccos} x = -\frac{1}{\sqrt{1-x^2}} \quad D \overline{\arctan} x = \frac{1}{1+x^2}$$

$$(f^{-1})'(y_0) = \frac{1}{f'(x_0)}, \text{ missä } y_0 = f(x_0)$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C (n \neq -1) \quad \int \frac{1}{x} dx = \ln |x| + C \quad \int \tan x dx = -\ln |\cos x| + C$$

$$\int \frac{dx}{\cos^2 x} = \int (1 + \tan^2 x) dx = \tan x + C \quad \int \frac{dx}{\sin^2 x} = \int (1 + \cot^2 x) dx = -\cot x + C$$

$$\int \frac{dx}{\sqrt{1-x^2}} = \overline{\arcsin} x + C \quad \int \frac{dx}{1+x^2} = \overline{\arctan} x + C$$

$$A = \int_a^b |f(x)| dx \quad V = \pi \int_a^b (f(x))^2 dx$$

$$Q(x) = (x-x_1)^{k_1} \dots (x-x_r)^{k_r} (x^2+c_1x+d_1)^{l_1} \dots (x^2+c_sx+d_s)^{l_s};$$

$$\frac{P(x)}{Q(x)} = \frac{A_{1,1}}{x-x_1} + \dots + \frac{A_{1,k_1}}{(x-x_1)^{k_1}} + \dots + \frac{A_{r,1}}{x-x_r} + \dots + \frac{A_{r,k_r}}{(x-x_r)^{k_r}}$$

$$+ \frac{B_{1,1}x + C_{1,1}}{x^2+c_1x+d_1} + \dots + \frac{B_{1,l_1}x + C_{1,l_1}}{(x^2+c_1x+d_1)^{l_1}} + \dots$$

$$+ \frac{B_{s,1}x + C_{s,1}}{x^2+c_sx+d_s} + \dots + \frac{B_{s,l_s}x + C_{s,l_s}}{(x^2+c_sx+d_s)^{l_s}}$$