TEKNILLINEN TIEDEKUNTA, MATEMATIIKAN JAOS

MATEMATIIKAN PERUSKURSSI I

3. välikoe 24.10.2013

VÄLIVAIHEET JA PERUSTELUT NÄKYVIIN, KIITOS!

1. a) Millä reaaliluvun x arvoilla epäyhtälö

$$\overline{\text{arc}}\sin(x-2) > 0$$

on määritelty? Ratkaise epäyhtälö. (2p)

  b) Laske seuraava raja-arvo käyttämättä derivaattaa:

$$\lim_{x\to 2}\frac{x^2-2x}{x^2-4}. \text{ (2p)}$$

  c) Laske seuraava raja-arvo käyttämättä derivaattaa:

$$\lim_{x\to 0}\frac{\sin(4x)}{\sqrt{2x+9}-3}. \text{ (2p)}$$

  d) Laske seuraava raja-arvo käyttämättä derivaattaa:

$$\lim_{x\to -\infty}(\sqrt{16x^2+5}-\sqrt{9x^2+5}). \text{ (2p)}$$

KAAVAKOKOELMA KÄÄNTÖPUOLELLA

$$\vec{u} \cdot \vec{v} = \sum_{i=1}^{3} u_i v_i \qquad \vec{u} \times \vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix} \qquad \vec{u} \times \vec{v} \cdot \vec{w} = \begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix}$$

$$d(P_1, l) = \frac{\|\vec{u} \times (\overrightarrow{AP_1})\|}{\|\vec{u}\|} \qquad d(P_1, T) = \frac{|\vec{n} \cdot (\overrightarrow{AP_1})|}{\|\vec{n}\|}$$

$$\sin^2 x + \cos^2 x = 1 \qquad \tan x = \frac{\sin x}{\cos x} \qquad \cot x = \frac{1}{\tan x}$$

$$\sin x = -\sin(-x) = \cos\left(\frac{\pi}{2} - x\right) \qquad \cos x = \cos(-x) = \sin\left(\frac{\pi}{2} - x\right)$$

$$\sin 2x = 2\sin x \cos x \qquad \cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$$

$$\frac{a}{\sin \alpha} = \frac{b}{\sin \beta} = \frac{c}{\sin \gamma} \qquad a^2 = b^2 + c^2 - 2bc\cos\alpha$$

$$\sin(x+y) = \sin x \cos y + \cos x \sin y \qquad \cos(x+y) = \cos x \cos y - \sin x \sin y$$

$$\sinh x = \frac{1}{2}(e^x - e^{-x}) \qquad \cosh x = \frac{1}{2}(e^x + e^{-x}) \qquad \tanh x = \frac{\sinh x}{\cosh x} \qquad \coth x = \frac{\cosh x}{\sinh x}$$

$$\cosh^2 x - \sinh^2 x = 1$$

$$Dx^n = nx^{n-1} \qquad D\sin x = \cos x \qquad D\cos x = -\sin x \qquad D\tan x = \frac{1}{\cos^2 x} = 1 + \tan^2 x$$

$$De^x = e^x \qquad Da^x = a^x \ln a (a > 0) \qquad D\ln|x| = \frac{1}{x} \qquad D\log_a |x| = \frac{1}{x \ln a}(a > 0, a \neq 1)$$

$$D\overline{\text{arc}}\sin x = \frac{1}{\sqrt{1-x^2}} \qquad D\overline{\text{arc}}\cos x = -\frac{1}{\sqrt{1-x^2}} \qquad D\overline{\text{arc}}\tan x = \frac{1}{1+x^2}$$

$$(f^{-1})'(y_0) = \frac{1}{f'(x_0)}, \text{ missä } y_0 = f(x_0) \qquad \kappa(t) = \frac{|x'(t)y''(t) - x''(t)y'(t)|}{[(x'(t))^2 + (y'(t))^2]^{\frac{3}{2}}}$$

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + C \ (n \neq -1) \qquad \int \frac{1}{x} \, dx = \ln|x| + C \qquad \int \tan x \, dx = -\ln|\cos x| + C$$

$$\int \frac{dx}{\cos^2 x} = \int (1 + \tan^2 x) \, dx = \tan x + C \qquad \int \frac{dx}{\sin^2 x} = \int (1 + \cot^2 x) \, dx = -\cot x + C$$

$$\int \frac{dx}{\sqrt{1-x^2}} = \overline{\text{arc}} \sin x + C \qquad \int \frac{dx}{1+x^2} = \overline{\text{arc}} \tan x + C$$

$$A = \int_a^b |f(x)| \, dx \qquad A = \frac{1}{2} \int_{\varphi_1}^{\varphi_2} (r(\varphi))^2 \, d\varphi$$

$$s = \int_a^b \sqrt{1 + (f'(x))^2} \, dx \qquad s = \int_{\varphi_1}^{\varphi_2} \sqrt{(r'(\varphi))^2 + (r(\varphi))^2} \, d\varphi \qquad s = \int_a^b \sqrt{(x'(t))^2 + (y'(t))^2} \, dt$$

$$V = \pi \int_a^b (f(x))^2 \, dx \qquad A = 2\pi \int_a^b f(x)\sqrt{1 + (f'(x))^2} \, dx$$

$$Q(x) = (x - x_1)^{k_1} \ldots (x - x_r)^{k_r} (x^2 + c_1 x + d_1)^{l_1} \ldots (x^2 + c_s x + d_s)^{l_s};$$

$$\frac{P(x)}{Q(x)} = \frac{A_{1,1}}{x - x_1} + \ldots + \frac{A_{1,k_1}}{(x - x_1)^{k_1}} + \ldots + \frac{A_{r,1}}{x - x_r} + \ldots + \frac{A_{r,k_r}}{(x - x_r)^{k_r}}$$

$$+ \frac{B_{1,1}x + C_{1,1}}{x^2 + c_1 x + d_1} + \ldots + \frac{B_{1,l_1}x + C_{1,l_1}}{(x^2 + c_1 x + d_1)^{l_1}} + \ldots$$

$$+ \frac{B_{s,1}x + C_{s,1}}{x^2 + c_s x + d_s} + \ldots + \frac{B_{s,l_s}x + C_{s,l_s}}{(x^2 + c_s x + d_s)^{l_s}}$$

ϕ	$\cos\phi$	$\sin\phi$
$\frac{\pi}{6}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$
$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$
$\frac{\pi}{3}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$