TEKNILLINEN TIEDEKUNTA, MATEMATIIKAN JAOS

MATEMATIIKAN PERUSKURSSI I

1. välikoe 26.9.2013

VÄLIVAIHEET JA PERUSTELUT NÄKYVIIN, KIITOS!

1. a) Ratkaise epäyhtälö

$$\sum_{k=2}^{3}(8-k^2)x^{k-1} > 0. \quad (2p)$$

b) Olkoot $\vec{u} = \vec{j} - 2\vec{k}$ ja $\vec{v} = 3\vec{i} - \vec{j}$ vektoriavaruuden $\mathbb{R}^3$ vektoreita. Laske $\vec{u} \times \vec{v}$ sekä tutki, ovatko vektorit $\vec{u} + \vec{v}$ ja $9\vec{u} - 4\vec{v}$ kohtisuorassa toisiaan vastaan. (2p)

c) Suora l kulkee suorien $l_1 : \vec{p} = t\vec{u} = t(3,-1,0)$, $t \in \mathbb{R}$, ja $l_2 : \vec{r} = \vec{b} + s\vec{v} = (0,2,-3) + s(2,0,-1)$, $s \in \mathbb{R}$, leikkauspisteen kautta ja sen suuntavektori on tason $T : 2x - y + 3z - 6 = 0$ normaalivektori $\vec{n}$. Tutki, onko piste $Q(0,-1,9)$ suoran l piste. (4p)

KAAVAKOKOELMA KÄÄNTÖPUOLELLA

$$\vec{u}\cdot\vec{v}=\sum_{i=1}^{3}u_iv_i \qquad \vec{u}\times\vec{v}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ u_1 & u_2 & u_3\\ v_1 & v_2 & v_3\end{vmatrix} \qquad \vec{u}\times\vec{v}\cdot\vec{w}=\begin{vmatrix}u_1 & u_2 & u_3\\ v_1 & v_2 & v_3\\ w_1 & w_2 & w_3\end{vmatrix}$$

$$d(P_1,l)=\frac{\|\vec{u}\times(\overrightarrow{AP_1})\|}{\|\vec{u}\|} \qquad d(P_1,T)=\frac{|\vec{n}\cdot(\overrightarrow{AP_1})|}{\|\vec{n}\|}$$

$$\sin^2 x+\cos^2 x=1 \qquad \tan x=\frac{\sin x}{\cos x} \qquad \cot x=\frac{1}{\tan x}$$

$$\sin x=-\sin(-x)=\cos\left(\frac{\pi}{2}-x\right) \qquad \cos x=\cos(-x)=\sin\left(\frac{\pi}{2}-x\right)$$

$$\sin 2x=2\sin x\cos x \qquad \cos 2x=\cos^2 x-\sin^2 x=2\cos^2 x-1=1-2\sin^2 x$$

$$\frac{a}{\sin\alpha}=\frac{b}{\sin\beta}=\frac{c}{\sin\gamma} \qquad a^2=b^2+c^2-2bc\cos\alpha$$

$$\sin(x+y)=\sin x\cos y+\cos x\sin y \qquad \cos(x+y)=\cos x\cos y-\sin x\sin y$$

$$\sinh x=\frac{1}{2}(e^x-e^{-x}) \qquad \cosh x=\frac{1}{2}(e^x+e^{-x}) \qquad \tanh x=\frac{\sinh x}{\cosh x} \qquad \coth x=\frac{\cosh x}{\sinh x}$$

$$\cosh^2 x-\sinh^2 x=1$$

$$Dx^n=nx^{n-1} \qquad D\sin x=\cos x \qquad D\cos x=-\sin x \qquad D\tan x=\frac{1}{\cos^2 x}=1+\tan^2 x$$

$$De^x=e^x \qquad Da^x=a^x\ln a(a>0) \qquad D\ln|x|=\frac{1}{x} \qquad D\log_a|x|=\frac{1}{x\ln a}(a>0,a\neq 1)$$

$$D\overline{\text{arc}}\sin x=\frac{1}{\sqrt{1-x^2}} \qquad D\overline{\text{arc}}\cos x=-\frac{1}{\sqrt{1-x^2}} \qquad D\overline{\text{arc}}\tan x=\frac{1}{1+x^2}$$

$$(f^{-1})'(y_0)=\frac{1}{f'(x_0)},\ \text{missä } y_0=f(x_0) \qquad \kappa(t)=\frac{|x'(t)y''(t)-x''(t)y'(t)|}{[(x'(t))^2+(y'(t))^2]^{\frac{3}{2}}}$$

$$\int x^n\,dx=\frac{x^{n+1}}{n+1}+C.(n\neq -1) \qquad \int\frac{1}{x}\,dx=\ln|x|+C \qquad \int\tan x\,dx=-\ln|\cos x|+C$$

$$\int\frac{dx}{\cos^2 x}=\int(1+\tan^2 x)\,dx=\tan x+C \qquad \int\frac{dx}{\sin^2 x}=\int(1+\cot^2 x)\,dx=-\cot x+C$$

$$\int\frac{dx}{\sqrt{1-x^2}}=\overline{\text{arc}}\ \sin x+C \qquad \int\frac{dx}{1+x^2}=\overline{\text{arc}}\ \tan x+C$$

$$A=\int_a^b|f(x)|\,dx \qquad A=\frac{1}{2}\int_{\varphi_1}^{\varphi_2}(r(\varphi))^2\,d\varphi$$

$$s=\int_a^b\sqrt{1+(f'(x))^2}\,dx \qquad s=\int_{\varphi_1}^{\varphi_2}\sqrt{(r'(\varphi))^2+(r(\varphi))^2}\,d\varphi \qquad s=\int_a^b\sqrt{(x'(t))^2+(y'(t))^2}\,dt$$

$$V=\pi\int_a^b(f(x))^2\,dx \qquad A=2\pi\int_a^b f(x)\sqrt{1+(f'(x))^2}\,dx$$

$$\begin{aligned}Q(x)&=(x-x_1)^{k_1}\dots(x-x_r)^{k_r}(x^2+c_1x+d_1)^{l_1}\dots(x^2+c_sx+d_s)^{l_s};\\ \frac{P(x)}{Q(x)}&=\frac{A_{1,1}}{x-x_1}+\dots+\frac{A_{1,k_1}}{(x-x_1)^{k_1}}+\dots+\frac{A_{r,1}}{x-x_r}+\dots+\frac{A_{r,k_r}}{(x-x_r)^{k_r}}\\ &+\frac{B_{1,1}x+C_{1,1}}{x^2+c_1x+d_1}+\dots+\frac{B_{1,l_1}x+C_{1,l_1}}{(x^2+c_1x+d_1)^{l_1}}+\dots\\ &+\frac{B_{s,1}x+C_{s,1}}{x^2+c_sx+d_s}+\dots+\frac{B_{s,l_s}x+C_{s,l_s}}{(x^2+c_sx+d_s)^{l_s}}\end{aligned}$$

ϕ	$\cos\phi$	$\sin\phi$
$\frac{\pi}{6}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$
$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$
$\frac{\pi}{3}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$