Tekniikan matematiikka

Kompleksianalyysi (031077P)

Loppukoe, 18.1.2016

1. a) Ratkaise yhtälö $z^4 = -16$. Anna ratkaisut z muodossa $z = x + \mathrm{i}y$.

   b) Määrää kaikki kompleksiluvut $z = x + \mathrm{i}y$, joille $e^z = -2 - 2\mathrm{i}$.

2. Määrää kaikki analyyttiset funktiot $f(z)$, joille $\operatorname{Re} f(z) = x^2 - y^2 - x + 2$.

3. Laske residylaskun avulla

$$\int_{-\infty}^{\infty} \frac{x^2}{x^4 + 5x^2 + 4}\, dx.$$

4. Diskreetin kausaalisen LTI-systeemin siirtofunktio on

$$H(z) = \frac{1}{3}\frac{z^2}{z^2 + z + 1}, \quad |z| > 1.$$

   Määrää systeemin impulssivaste $h(k)$.

Kaavoja:

$$f^{(n)}(z_0) = \frac{n!}{2\pi \mathrm{i}} \int\limits_C \frac{f(z)}{(z-z_0)^{n+1}}\, dz$$

$$a_{-1} = \frac{1}{(n-1)!}\left[\frac{d^{n-1}}{dz^{n-1}}(z-z_0)^n f(z)\right]_{z=z_0}$$

$$\frac{1}{2\pi \mathrm{i}} \int\limits_C \frac{f'(z)}{f(z)}\, dz = M - N$$

$$H(\omega) = \sum_n h(n) e^{-\mathrm{i}\omega n}$$

$$X(\omega) = \sum_n x(n) e^{-\mathrm{i}\omega n}$$

$$x(n-k) \leftrightarrow X(\omega) e^{-\mathrm{i}\omega k}$$

$$Y(\omega) = H(\omega) X(\omega)$$

$$h(k) = \frac{1}{2\pi \mathrm{i}} \int\limits_{S_r} H(z) z^{k-1} dz$$

$$H(z) = \sum_{k=0}^{\infty} h(k) z^{-k}$$

$$\sin z = \frac{e^{\mathrm{i}z} - e^{-\mathrm{i}z}}{2\mathrm{i}}, \cos z = \frac{e^{\mathrm{i}z} + e^{-\mathrm{i}z}}{2}$$

$$e^{\mathrm{i}z} = \cos z + \mathrm{i} \sin z$$

$$w = A\frac{z+B}{z+C}$$

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}, \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

$$\sin(z_1 + z_2) = \sin z_1 \cos z_2 + \cos z_1 \sin z_2$$

$$\cos(z_1 + z_2) = \cos z_1 \cos z_2 - \sin z_1 \sin z_2$$

$$R = \lim_{n\to\infty} \left|\frac{c_n}{c_{n+1}}\right|$$

$$\begin{aligned}
& B(z) \text{ Schur, jos } 1)|b_0| < |b_n| \text{ ja } 2)\ B_1(z) \text{ Schur} \\
B(z) &= b_n z^n + b_{n-1} z^{n-1} + \cdots + b_1 z + b_0,\ B_1(z) = a_0 + a_1 z + \cdots + a_{n-1} z^{n-1} \\
a_0 &= \begin{vmatrix} b_n & b_{n-1} \\ b_0 & b_1 \end{vmatrix},\ a_1 = \begin{vmatrix} b_n & b_{n-2} \\ b_0 & b_2 \end{vmatrix}, \cdots, a_{n-1} = \begin{vmatrix} b_n & b_0 \\ b_0 & b_n \end{vmatrix}
\end{aligned}$$

$$\frac{w-w_1}{w-w_2} \cdot \frac{w_3-w_2}{w_3-w_1} = \frac{z-z_1}{z-z_2} \cdot \frac{z_3-z_2}{z_3-z_1}.$$