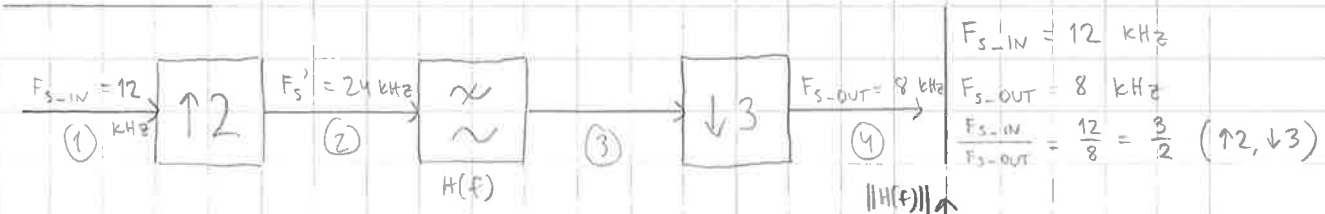
DIGITAL FILTERS - WEEK EXAM 8

PROBLEM 1



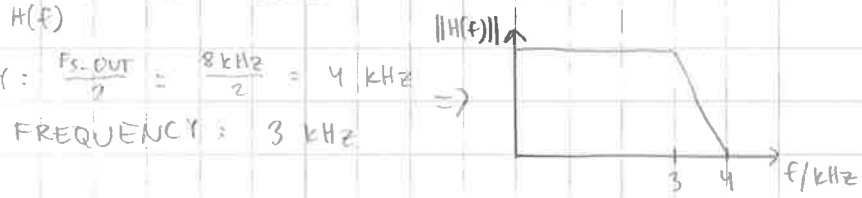
$F_{s\_IN} = 12$ kHz

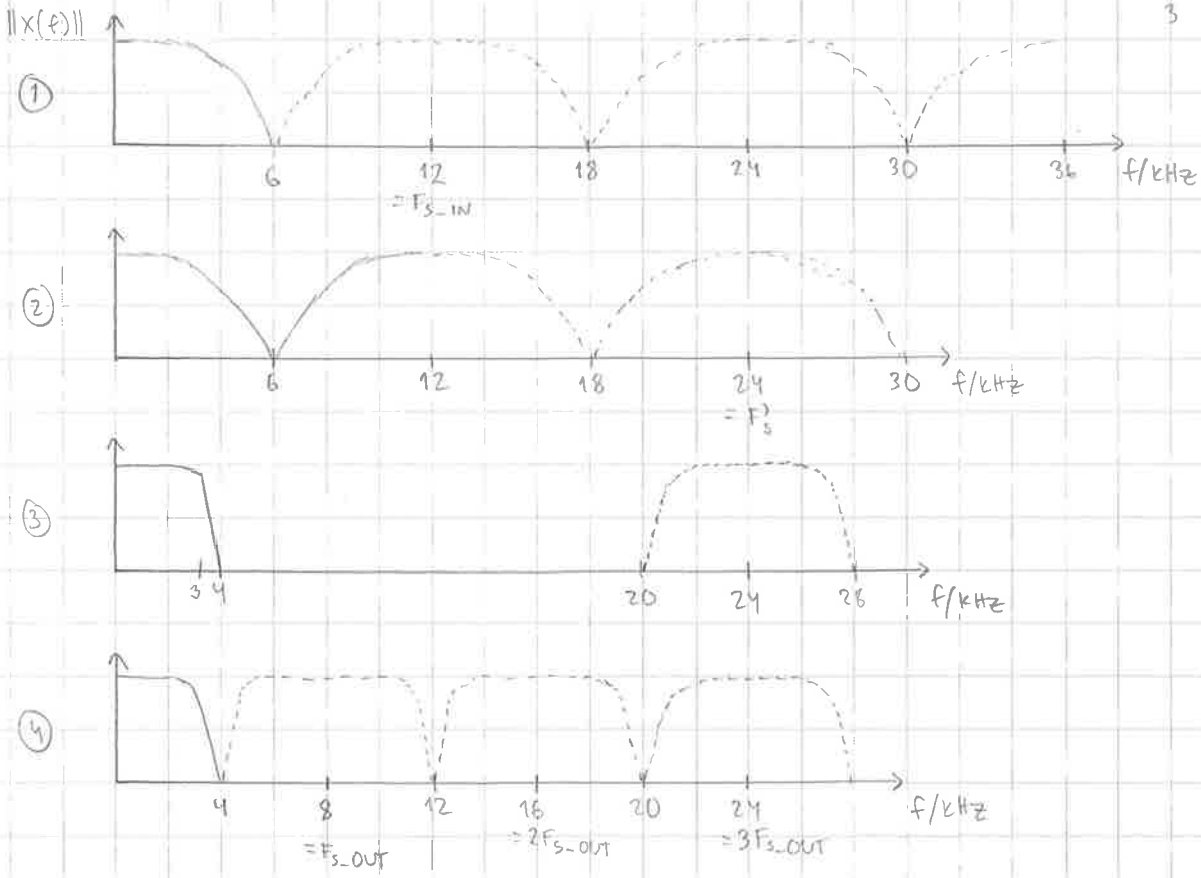
$F_{s\_OUT} = 8$ kHz

$\frac{F_{s\_IN}}{F_{s\_OUT}} = \frac{12}{8} = \frac{3}{2}$ $(\uparrow 2, \downarrow 3)$

FINAL NYQUIST FREQUENCY: $\frac{F_{s\_OUT}}{2} = \frac{8\text{ kHz}}{2} = 4$ kHz

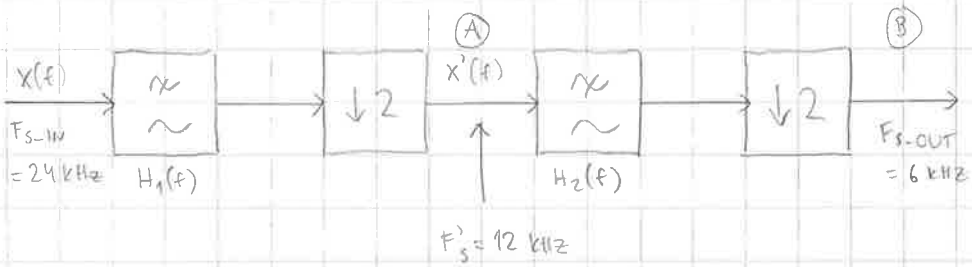
INTERESTING BAND EDGE FREQUENCY: 3 kHz





PROBLEM 2

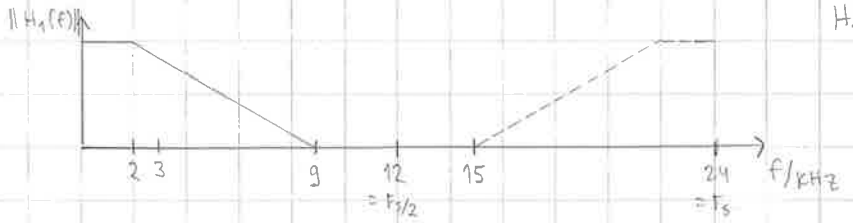
$\frac{F_{s\_IN} = 24\ kHz}{F_{s\_OUT} = 6\ kHz}$; $\frac{F_{s\_IN}}{F_{s\_OUT}} = \frac{24\ kHz}{6\ kHz} = 4 = 2 \cdot 2 \quad (\downarrow 2, \downarrow 2)$



ORIGINAL SIGNAL:

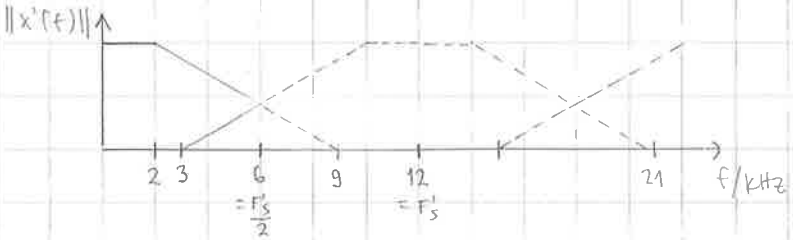
BOTH FILTERS NEED TO PREVENT ALIASING BELOW THE NYQUIST FREQ. OF THE FINAL SAMPLING RATE : $\frac{F_{s\_OUT}}{2} = \frac{6\ kHz}{2} = 3\ kHz$

$H_1(f)$ OPERATES AT $F_S = 24\ kHz$



$H_1(f)$: TRANSITION BAND
9 kHz − 2 kHz = 7 kHz

AT POINT (A)

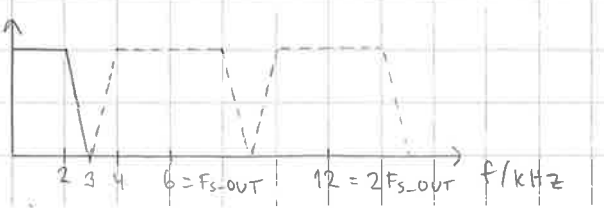


WE CAN ALLOW ALIASING UNTIL 3 kHz, BUT NOT LOWER! (WIDER TRANSITION BAND TO KEEP FILTER LENGTH AND COST LOW!)

$H_2(f)$ OPERATES AT $F'_S = 12\ kHz$

TRANSITION BAND DESIGNED SO THAT NO ALIASING BELOW 3 kHz AFTER 12 kHz → 6 kHz

AT POINT (B) (OUTPUT) $F_{s\_OUT} = 6\ kHz$



FILTER SPECS:

$H_1(f)$: $f_p = 2$ kHz

$f_s = 9$ kHz

$F_s = 24$ kHz

$\Delta f_1 = \frac{f_s - f_p}{F_s} = \frac{(9-2)\,\text{kHz}}{24\,\text{kHz}} = 0{,}292$

$H_2(f)$: $f_p = 2$ kHz

$f_s = 3$ kHz

$F_s' = 12$ kHz

$\Delta f_2 = \frac{f_s - f_p}{F_s'} = \frac{(3-2)\,\text{kHz}}{12\,\text{kHz}} = 0{,}083$

STOPBAND ATTENUATION $A_s \geq 48$ dB => AT LEAST HAMMING

PASSBAND RIPPLE CAN INCREASE AT MOST 0,1 dB

$$A_p = 20 \log(1+\delta_p) \leq 0{,}1 \text{ dB}$$

$$\log(1+\delta_p) \leq \frac{0{,}1}{20}$$

$$\delta_p \leq 10^{\frac{0{,}1}{20}} - 1 = 0{,}011579$$

DIVIDE δ_p EQUALLY FOR EACH FILTER

$$\delta_{p1} = \delta_{p2} = \delta_p/2 = 0{,}00579$$

$$A_{p1} \leq 20 \log(1+\delta_{p1}) = 20 \log(1+0{,}00579) = 0{,}050 \text{ dB} = A_{p2}$$

$A_{p1} = A_{p2} \leq 0{,}05$ dB => HAMMING SUFFICES

$$\Delta f = \frac{3{,}3}{N} \Rightarrow N = \frac{3{,}3}{\Delta f}$$

FILTER LENGTHS

$H_1(f)$: $N_1 = \frac{3{,}3}{\Delta f_1} = \frac{3{,}3}{0{,}292} = 11{,}3 \approx 12$

$H_2(f)$: $N_2 = \frac{3{,}3}{\Delta f_2} = \frac{3{,}3}{0{,}083} = 39{,}76 \approx 40$

COMPUTING COSTS: $F_{s1} \cdot N_1 + F_{s2} \cdot N_2 = 24\,\text{kHz} \cdot 12 + 12\,\text{kHz} \cdot 40$

$= 768 \cdot 10^3$ MACS