DIGITAL FILTERS - WEEK EXAM 7

PROBLEM 1

PASSBAND $f_p = 500$ Hz $= f_c$

STOPBAND $f_s = 6{,}25$ kHz

$A_s \geq 40$ dB

$F_s = 25$ kHz

ATTENUATION AT STOPBAND EDGE FREQUENCY 6,25 kHz HAS TO BE AT LEAST 40 dB

$$BW: \|H(f)\| = \frac{1}{\sqrt{1+\left(\frac{f}{f_c}\right)^{2n}}}$$

DETERMINE THE LOWEST POSSIBLE ORDER FOR THE BUTTERWORTH LPF TO MEET THE REQUIREMENTS

$$-20\log\|H(6{,}25\text{ kHz})\| \geq 40\text{ dB}$$

$$-20\log\left(\frac{1}{\sqrt{1+\left(\frac{6{,}25\text{ kHz}}{0{,}5\text{ kHz}}\right)^{2n}}}\right) \geq 40\text{ dB}$$

$$-20\log\left(1+\left(\frac{6{,}25}{0{,}5}\right)^{2n}\right)^{-\frac{1}{2}} \geq 40\text{ dB}$$

LOGARITHM RULES

$$10\log\left(1+\left(\frac{6{,}25}{0{,}5}\right)^{2n}\right) \geq 40$$

$$\log\left(1+\left(\frac{6{,}25}{0{,}5}\right)^{2n}\right) \geq 4$$

$$\log_{10}(1+x) = 4 \Rightarrow 1+x = 10^4$$
$$x = 10^4 - 1$$

$$\left(\frac{6{,}25}{0{,}5}\right)^{2n} \geq 10^4 - 1$$

$$\log\left(\frac{6{,}25}{0{,}5}\right)^{2n} \geq \log(10^4-1)$$

$$2n\log\left(\frac{6{,}25}{0{,}5}\right) \geq \log(10^4-1)$$

$$n \geq \frac{\log(10^4-1)}{2\log\left(\frac{6{,}25}{0{,}5}\right)} = 1{,}823 \Rightarrow \underline{\underline{n=2}}$$

SECOND ORDER BUTTERWORTH APPROXIMATION

$$H(s) = \frac{1}{1+\sqrt{2}\,s+s^2}$$

NEXT, PREWARPING AND BZT!

PREWARPING $(k=1)$

$$\omega'_p = \tan\left(\frac{\omega_p T}{2}\right) = \tan\left(\frac{\omega_p}{2F_s}\right) = \tan\left(\frac{2\pi \cdot 500}{2 \cdot 25000}\right) = 0{,}062915$$

$$\approx 0{,}063$$

THE APPLICATION NEEDS A LOW-PASS FILTER => LPF → LPF

$$s = \frac{s}{\omega'_p} = \frac{s}{0{,}063}$$

EXPAND BY $0{,}063^2$

$$H\left(\frac{s}{0{,}063}\right) = \frac{1}{1 + \frac{\sqrt{2}\,s}{0{,}063} + \frac{s^2}{0{,}063^2}} = \frac{0{,}063^2}{0{,}063^2 + 0{,}063 \cdot \sqrt{2} \cdot s + s^2}$$

$$= \frac{0{,}00397}{0{,}00397 + 0{,}089\,s + s^2}$$

BZT: $s = \frac{z-1}{z+1}$ $(k=1)$

$$H\left(\frac{z-1}{z+1}\right) = \frac{0{,}00397}{0{,}00397 + 0{,}089\,\frac{z-1}{z+1} + \left(\frac{z-1}{z+1}\right)^2}$$ EXPAND BY $(z+1)^2$

$$= \frac{0{,}00397\,(z+1)^2}{0{,}00397 \cdot (z+1)^2 + 0{,}089 \cdot (z-1)(z+1) + (z-1)^2}$$

$$= \frac{0{,}00397\,(z^2+2z+1)}{0{,}00397\,(z^2+2z+1) + 0{,}089\,(z^2-1) + z^2 - 2z + 1}$$

$$= \frac{0{,}00397\,(z^2+2z+1)}{(0{,}00397 + 0{,}089 + 1) \cdot z^2 + (2 \cdot 0{,}00397 - 2)\,z + (0{,}00397 - 0{,}089 + 1)}$$

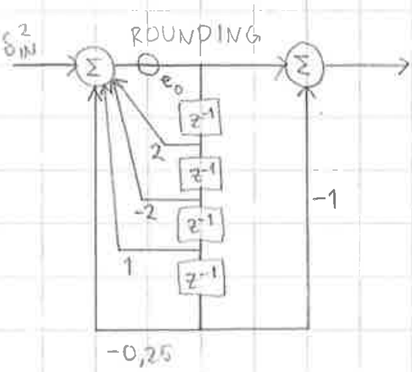
$$= \frac{0{,}00397\,(z^2+2z+1)}{1{,}09297\,z^2 - 1{,}99206\,z + 0{,}91497}$$

$$\approx \frac{0{,}00397\,(z^2+2z+1)}{1{,}093\,z^2 - 1{,}992\,z + 0{,}915}$$

PROBLEM 2

$$H(z) = \frac{1 - z^{-4}}{1 - 2z^{-1} + 2z^{-2} - z^{-3} + 0{,}25 z^{-4}} = \underbrace{\frac{1 - z^{-2}}{1 - z^{-1} + 0{,}5 z^{-2}}}_{H_1(z)} \cdot \underbrace{\frac{1 + z^{-2}}{1 - z^{-1} + 0{,}5 z^{-2}}}_{H_2(z)}$$

i) δ_{IN}^2



THE ROUNDING SPOT e_0 "SEES" THE ENTIRE TRANSFER FUNCTION $H(z)$

→ CALCULATE THE IMPULSE RESPONSE $h(n)$

NOISE POWER $e^2(n) = \frac{q^2}{12}$ (SAME BIT PRECISION)

NOISE POWER IN THE INPUT $\delta_{IN}^2 = \frac{q^2}{12}$ (NOT REQUIRED)

$$\begin{array}{l}
\qquad\qquad\qquad\qquad\qquad 1 + 2z^{-1} + 2z^{-2} \\
1 - 2z^{-1} + 2z^{-2} - z^{-3} + 0{,}25 z^{-4} \;\big|\; 1 + 0z^{-1} + 0z^{-2} + 0z^{-3} + z^{-4} \\
\qquad -\left(1 - 2z^{-1} + 2z^{-2} - z^{-3} + 0{,}25 z^{-4}\right) \\
\qquad\qquad 2z^{-1} - 2z^{-2} + z^{-3} - 1{,}25 z^{-4} + 0z^{-5} \\
\qquad -\left(2z^{-1} - 4z^{-2} + 4z^{-3} - 2z^{-4} + 0{,}5 z^{-5}\right) \\
\qquad\qquad 2z^{-2} - 3z^{-3} + 0{,}75 z^{-4} - 0{,}5 z^{-5} \\
\qquad\qquad \dots
\end{array}$$

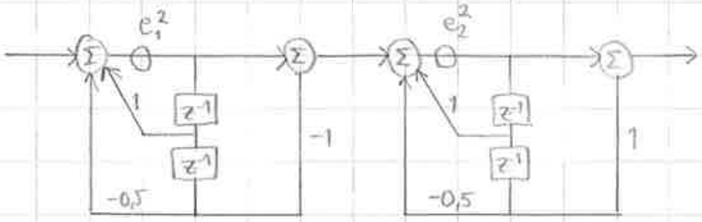
$\rightarrow h(n) = \{1, 2, 2, \dots\}$

$\rightarrow \sum h^2(n) = 1^2 + 2^2 + 2^2 = 9$

TOTAL OUTPUT NOISE POWER:

$$\delta_{OUT}^2 = \delta_{IN}^2 \cdot \sum h^2(n) + e^2 \cdot \sum h^2(n) = \frac{q^2}{12} \cdot 9 + \frac{q^2}{12} \cdot 9 = \frac{q^2}{12} \cdot 9 \cdot 2 = \underline{\underline{18 \frac{q^2}{12}}}$$

ii)



- δ_{IN}^2 AND e_1^2 SEE THE ENTIRE CASCADE TRANSFER FUNCTION $H_1(z) \cdot H_2(z) = H(z)$ $(h(n))$
- e_2^2 SEES ONLY $H_2(z)$

$$\begin{array}{l}
\qquad\qquad\qquad 1 + z^{-1} + 1{,}5 z^{-2} \\
1 - z^{-1} + 0{,}5 z^{-2} \;\big|\; 1 + 0z^{-1} + z^{-2} \\
\qquad -\left(1 - z^{-1} + 0{,}5 z^{-2}\right) \\
\qquad\qquad z^{-1} + 0{,}5 z^{-2} \\
\qquad -\left(z^{-1} - z^{-2} + 0{,}5 z^{-3}\right) \\
\qquad\qquad +1{,}5 z^{-2} - 0{,}5 z^{-3} \\
\qquad\qquad \dots
\end{array}$$

$\rightarrow h_2(n) = \{1, 1, 1{,}5, \dots\}$

$\rightarrow \sum h_2^2(n) = 1^2 + 1^2 + 1{,}5^2 = 4{,}25$

TOTAL OUTPUT NOISE POWER: (CASCADE)

$$\begin{aligned}
\delta_{OUT-C}^2 &= \delta_{IN}^2 \cdot \sum h^2(n) + e_1^2 \cdot \sum h^2(n) + e_2^2 \cdot \sum h_2^2(n) \\
&= \frac{q^2}{12} \cdot 9 + \frac{q^2}{12} \cdot 9 + \frac{q^2}{12} \cdot 4{,}25 \\
&= \frac{q^2}{12} \cdot 9 \cdot 2 + \frac{q^2}{12} \cdot 4{,}25 = \underline{\underline{22{,}5 \frac{q^2}{12}}}
\end{aligned}$$

DIFFERENCE: $10 \log\left(22{,}5 \frac{q^2}{12}\right) - 10 \log\left(18 \frac{q^2}{12}\right) = 10 \log\left(\frac{22{,}5 \frac{q^2}{12}}{18 \frac{q^2}{12}}\right) = \underline{\underline{0{,}97 \text{ dB}}}$