DIGITAL FILTERS - WEEK EXAM 6

PROBLEM 1

$h(n) = [-0{,}0977 \quad 0{,}3819 \quad 0{,}5186 \quad 0{,}3819 \quad -0{,}0977]$

$F_s = 2000$ Hz $\quad (= 2\pi)$

INTERESTING FREQUENCY 1000 Hz $\quad (= \pi)$

QUANTIZE TO 4 BITS, VALUE RANGE $[-1, 1) \quad | = [-A, A)$

$q = \frac{2A}{2^B - 1} \approx \frac{2A}{2^B} = \frac{2}{2^4} = \frac{1}{8} = 0{,}125$ (QUANTIZATION INTERVAL)

QUANTIZED VALUES $h_q(n)$: (MANY WAYS TO COMPUTE ...)

$h_q(n) = \text{int}\left(\frac{h(n)}{q} + 0{,}5\right) \cdot q \qquad |\ \text{int}(x + 0{,}5)$ is basically round(x)

$h_q(0) = \text{int}\left(\frac{-0{,}0977}{0{,}125} + 0{,}5\right) \cdot 0{,}125 = -1 \cdot 0{,}125 = -0{,}125 \qquad (= h_q(4))$

$h_q(1) = \text{int}\left(\frac{0{,}3819}{0{,}125} + 0{,}5\right) \cdot 0{,}125 = 3 \cdot 0{,}125 = 0{,}375 \qquad (= h_q(3))$

$h_q(2) = \text{int}\left(\frac{0{,}5186}{0{,}125} + 0{,}5\right) \cdot 0{,}125 = 4 \cdot 0{,}125 = 0{,}5$

$h_{q4} = [-0{,}125 \quad 0{,}375 \quad 0{,}5 \quad 0{,}375 \quad -0{,}125]$

$H(z) = h(0) + h(1)z^{-1} + h(2)z^{-2} + h(3)z^{-3} + h(4)z^{-4}$

ORIGINAL MAGNITUDE RESPONSE AT π $\quad (z = e^{j\pi})$

$$H(e^{j\pi}) = -0{,}0977 + 0{,}3819\,\overbrace{e^{-j\pi}}^{=-1} + 0{,}5186\,\overbrace{e^{-j2\pi}}^{=1} + 0{,}3819\,\overbrace{e^{-j3\pi}}^{=-1} - 0{,}0977\,\overbrace{e^{-j4\pi}}^{=1}$$

$$= 2 \cdot (-0{,}0977) - 2 \cdot 0{,}3819 + 0{,}5186$$

$$= -0{,}4406 \quad ; \quad 20 \cdot \log_{10}(0{,}4406) = -7{,}12 \text{ dB}$$

QUANTIZED MAGNITUDE RESPONSE AT π

$$H_{q4}(e^{j\pi}) = -0{,}125 + 0{,}375\,e^{-j\pi} + 0{,}5\,e^{-j2\pi} + 0{,}375\,e^{-j3\pi} - 0{,}125\,e^{-j4\pi}$$

$$= 2 \cdot (-0{,}125) - 2 \cdot 0{,}375 + 0{,}5$$

$$= -0{,}5 \quad ; \quad 20 \log_{10}(0{,}5) = -6{,}02 \text{ dB}$$

STOPBAND ATTENUATION DECREASE AT 1000 Hz:

$7{,}12 \text{ dB} - 6{,}02 \text{ dB} = 1{,}1 \text{ dB}$

1,1 dB DECREASE IN ATTENUATION AT 1000 Hz

PROBLEM 2

$$H(z) = \frac{1,2z}{z-e^{-1,8}} + \frac{2z}{z-e^{-1,5}}$$

CHANGE TRANSFER FUNCTION TO CAUSAL FORM

$$H(z) = \frac{1,2}{1-e^{-1,8}z^{-1}} + \frac{2}{1-e^{-1,5}z^{-1}}$$

NOTICE AS SUMS OF GEOMETRIC SERIES

$$H(z) = 1,2\sum_{n=0}^{\infty}\left(e^{-1,8}z^{-1}\right)^n + 2\sum_{n=0}^{\infty}\left(e^{-1,5}z^{-1}\right)^n$$

NOW YOU REMEMBER THAT $H(s) = \frac{C}{s+p}$ HAS THE INVERSE TRANSFORM $L^{-1}(H(s)) = Ce^{-pt}$ AND $H(z) = \sum_{n=0}^{\infty} C e^{-pnT} z^{-n} = \sum_{n=0}^{\infty} C\left(e^{-pT}z^{-1}\right)^n$

$$H(z) = 1,2\sum_{n=0}^{\infty}\left(e^{-1,8}z^{-1}\right)^n + 2\sum_{n=0}^{\infty}\left(e^{-1,5}z^{-1}\right)^n \quad \Big| \quad C\sum_{n=0}^{\infty}\left(e^{-pT}z^{-1}\right)^n \qquad (T=1)$$

$$\Rightarrow h(t) = 1,2e^{-1,8t} + 2e^{-1,5t} \qquad \left(= L^{-1}(H(s))\right)$$

$$\Rightarrow L(h(t)) = \underline{\underline{H(s) = \frac{1,2}{s+1,8} + \frac{2}{s+1,5}}}$$

PROBLEM 1 ALTERNATIVELY

$h_{q4}(n) = [-0{,}125 \;\; 0{,}375 \;\; 0{,}5 \;\; 0{,}375 \;\; -0{,}125]$

$h(n) = [-0{,}0977 \;\; 0{,}3819 \;\; 0{,}5186 \;\; 0{,}3819 \;\; -0{,}0977]$

$e_4(n) = [0{,}0273 \;\; 0{,}0069 \;\; 0{,}0186 \;\; 0{,}0069 \;\; 0{,}0273]$ | $e_4(n) = h(n) - h_{q4}(n)$ ERROR

ERROR MAGNITUDE RESPONSE AT π

$$E(e^{j\pi}) = 0{,}0273 + 0{,}0069\underbrace{e^{-j\pi}}_{=-1} + 0{,}0186\underbrace{e^{-j2\pi}}_{=1} + 0{,}0069\underbrace{e^{-j3\pi}}_{=-1} + 0{,}0273\underbrace{e^{-j4\pi}}_{=1}$$

$$= 2\cdot 0{,}0273 - 2\cdot 0{,}0069 + 0{,}0186$$

$$= 0{,}0594$$

STOPBAND ATTENUATION DECREASE AT 1000 Hz:

$$\underbrace{-20\log_{10}(0{,}4406)}_{\text{ORIGINAL}} - \Big(\underbrace{-20\log_{10}(0{,}4406 + 0{,}0594)}_{\text{ORIGINAL + ERROR}}\Big) = 7{,}12\text{ dB} - 6{,}02\text{ dB}$$

$$= \underline{\underline{1{,}1\text{ dB}}}$$

SAME RESULT!