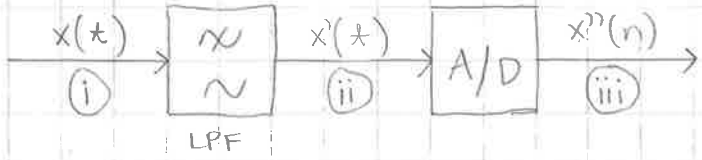
DIGITAL FILTERS - WEEK EXAM 1

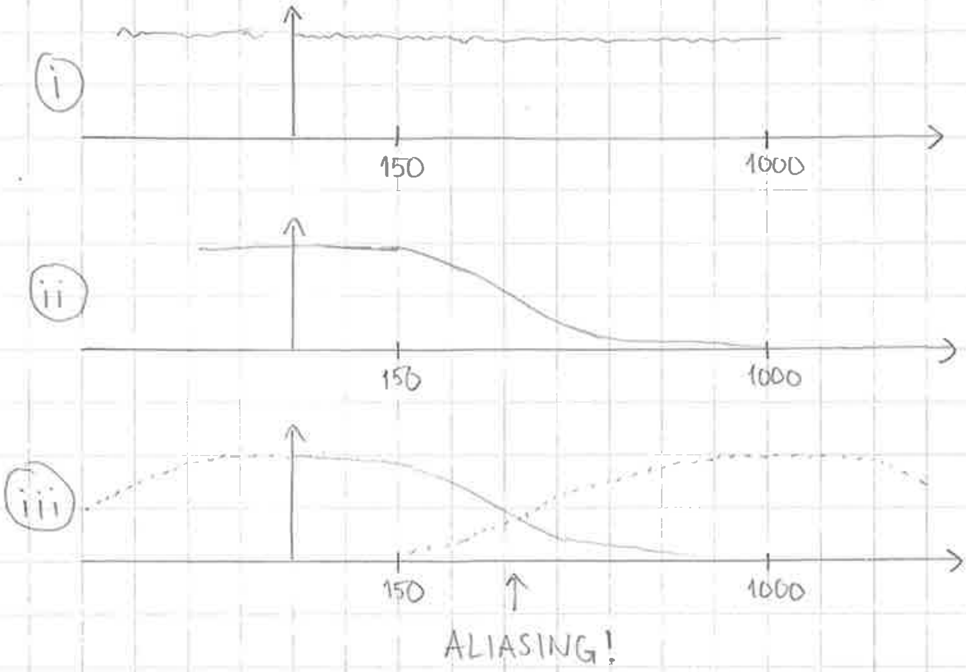
TASK 1

Interesting band $[0, 150]$ Hz

$F_s = 1000$ Hz

$f_c = ?$, $n = ?$





8 BIT SQNR : $SQNR_8 = (6{,}02 \cdot 8 + 1{,}76)\,dB = 49{,}9\ dB$

MAX ATTENUATION 1 dB AT 150 Hz

$$\|H(f)\| = \frac{1}{\sqrt{1+\left(\frac{f}{f_c}\right)^{2n}}} \xRightarrow{\text{IN dB}} 20\log\left(\frac{1}{\sqrt{1+\left(\frac{f}{f_c}\right)^{2n}}}\right)$$

$$\begin{cases} 20\log\left(\dfrac{1}{\sqrt{1+\left(\frac{150}{f_c}\right)^{2n}}}\right) \geq -1\ dB & \text{(a)} \qquad 150\ Hz \\[2ex] 20\log\left(\dfrac{1}{\sqrt{1+\left(\frac{850}{f_c}\right)^{2n}}}\right) \leq -49{,}9\ dB & \text{(b)} \qquad (1000-150)\,Hz = 850\ Hz \end{cases}$$

a) $$20\log\left(\frac{1}{\sqrt{1+\left(\frac{150}{f_c}\right)^{2n}}}\right) \geq -1\,dB$$

$$20\log\left(1+\left(\frac{150}{f_c}\right)^{2n}\right)^{-\frac{1}{2}} \geq -1$$

$$-10\log\left(1+\left(\frac{150}{f_c}\right)^{2n}\right) \geq -1$$

$$\log\left(1+\left(\frac{150}{f_c}\right)^{2n}\right) \leq 0{,}1$$

$$\left(\frac{150}{f_c}\right)^{2n} \leq 10^{0{,}1}-1$$

b) $$20\log\left(\frac{1}{\sqrt{1+\left(\frac{850}{f_c}\right)^{2n}}}\right) \leq -49{,}9\,dB$$

$$20\log\left(1+\left(\frac{850}{f_c}\right)^{2n}\right)^{-\frac{1}{2}} \leq -49{,}9$$

$$-10\log\left(1+\left(\frac{850}{f_c}\right)^{2n}\right) \leq -49{,}9$$

$$\log\left(1+\left(\frac{850}{f_c}\right)^{2n}\right) \geq 4{,}99$$

$$\left(\frac{850}{f_c}\right)^{2n} \geq 10^{4{,}99}-1$$

SYSTEM OF EQUATIONS AND LOGARITHM RULES

$$\begin{cases}\left(\frac{150}{f_c}\right)^{2n} = 10^{0{,}1}-1 \\ \left(\frac{850}{f_c}\right)^{2n} = 10^{4{,}99}-1\end{cases} \Rightarrow \log\left(\frac{150}{850}\right)^{2n} = \log\left(\frac{10^{0{,}1}-1}{10^{4{,}99}-1}\right)$$

$$2n\cdot\log\left(\frac{150}{850}\right) = \log\left(\frac{10^{0{,}1}-1}{10^{4{,}99}-1}\right)$$

$$n = \frac{\log\left(\frac{10^{0{,}1}-1}{10^{4{,}99}-1}\right)}{2\cdot\log\left(\frac{150}{850}\right)} = \frac{-5{,}5768}{-1{,}5067} = 3{,}7 \Rightarrow \underline{\underline{n=4}}$$

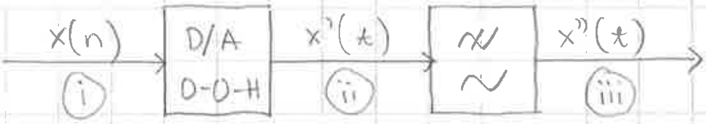
f_c: $$\left(\frac{150}{f_c}\right)^{2n} = 10^{0{,}1}-1$$

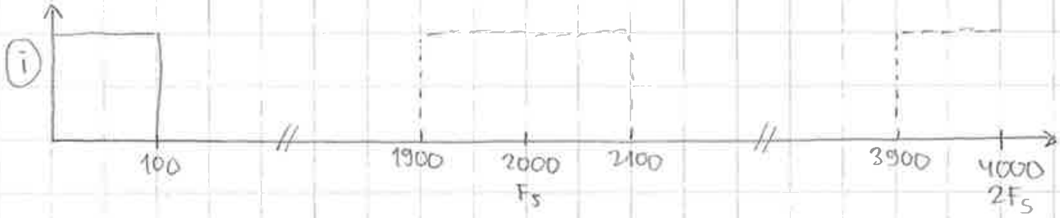
$$\left(\frac{150}{f_c}\right)^{8} = 10^{0{,}1}-1$$

$$\frac{150}{f_c} = \sqrt[8]{10^{0{,}1}-1}$$

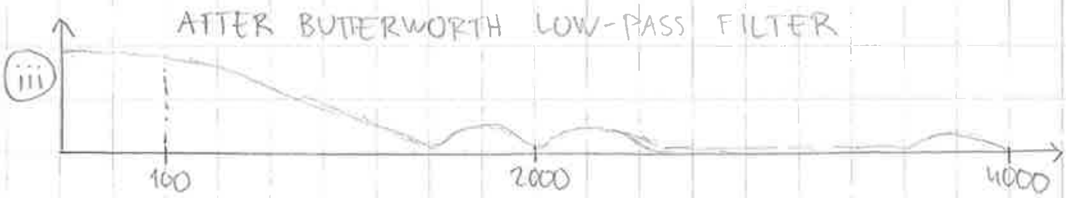
$$f_c = \frac{150}{\sqrt[8]{10^{0{,}1}-1}} = \underline{\underline{177{,}6\ Hz}}$$

TASK 2





AFTER BUTTERWORTH LOW-PASS FILTER



WHAT ARE WE INTERESTED IN ?

- SINC AT 100 Hz (interesting band!)
- SINC AT 1900 Hz

$$SQNR_8 = (6{,}02 \cdot 8 + 1{,}76)\,dB = 49{,}9\ dB$$

SINC: $\dfrac{\sin\left(\frac{\omega T}{2}\right)}{\frac{\omega T}{2}} = \dfrac{\sin\left(\frac{2\pi f}{2F_s}\right)}{\frac{2\pi f}{2F_s}} = \dfrac{\sin\left(\frac{\pi f}{F_s}\right)}{\frac{\pi f}{F_s}}$

SINC 100 Hz:

$$20\log\left(\frac{\sin\left(\frac{\pi \cdot 100}{2000}\right)}{\frac{\pi \cdot 100}{2000}}\right) = -0{,}035749\ dB$$

SINC 1900 Hz:

$$20\log\left(\frac{\sin\left(\frac{\pi \cdot 1900}{2000}\right)}{\frac{\pi \cdot 1900}{2000}}\right) = -25{,}6\ dB$$

ATTENUATION REQUIREMENT FOR BUTTERWORTH LPF AT 1900 Hz:

$$(49{,}9 - 25{,}6)\ dB = 24{,}3\ dB$$

$$20\log\left(\frac{1}{\sqrt{1+\left(\frac{1900}{f_c}\right)^4}}\right) = -24{,}3 \text{ dB} \qquad (n=2)$$

$$20\log\left(1+\left(\frac{1900}{f_c}\right)^4\right)^{-\frac{1}{2}} = -24{,}3$$

$$-10\log\left(1+\left(\frac{1900}{f_c}\right)^4\right) = 2{,}43$$

$$\left(\frac{1900}{f_c}\right)^4 = 10^{2{,}43}-1$$

$$\frac{1900}{f_c} = \sqrt[4]{10^{2{,}43}-1}$$

$$f_c = \frac{1900}{\sqrt[4]{10^{2{,}43}-1}} = 469{,}524 \text{ Hz}$$

$$\approx \underline{\underline{470 \text{ Hz}}}$$

BW LPF AT 100 Hz :

$$20\log\left(\frac{1}{\sqrt{1+\left(\frac{100}{470}\right)^4}}\right) = -0{,}008891 \text{ dB}$$

TOTAL ATTENUATION AT 100 Hz :

$$-(0{,}00889 + 0{,}03575)\text{ dB} = \underline{\underline{-0{,}04464 \text{ dB}}}$$