DIGITAL FILTERS 521337A
Exam 02.04.2004

YOU ARE ALLOWED TO BRING ONE A4-SIZE PAPER FILLED (BOTH SIDES CAN BE USED) WITH FORMULAS AND OTHER INFORMATION.

1. In Figure 1 is a spectrum of an analog signal x(t). The spectrum is symmetric with respect to the zero-frequency. The signal is converted into digital form using 7 kHz sampling frequency. The signal is filtered before A/D- conversion using analog Butterworth filter to reduce aliasing problems. The amplitude response of the Butterworth filter is

$$|H(f)| = \frac{1}{\sqrt{1+\left(\frac{f}{f_c}\right)^{2n}}}.$$

   a) Draw the spectrum of the signal, which is sampled without anti-aliasing filter. Use the frequency range [0, 7 kHz]. (1p)
   b) What is the degree n of the needed Butterworth filter, which produces attenuation of 3 dB at frequency 1 kHz and attenuates the aliasing frequencies at least 40 dB <u>compared to the edge of passband</u>? (2p)
   c) What is the lowest sampling frequency, which can be used to A/D-convert the signal x(t) without aliasing error? Draw the signal, which is sampled using this frequency into a diagram. Use normalized frequency scale [0, 1]. (1p)

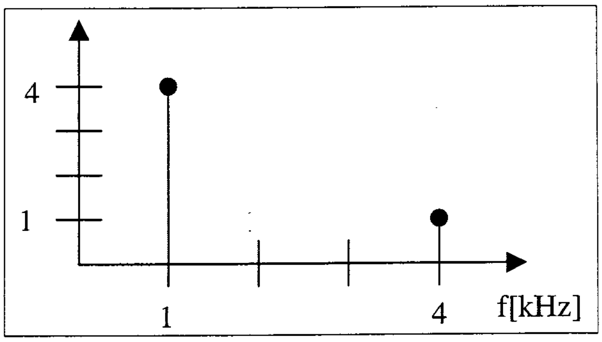

Figure 1.

2. The difference equation of a digital filter is
   $y(n) = 2x(n) - 4x(n+1) + 2x(n+2) - 2y(n+1) - 2y(n+2)$.
   a) Is it IIR – or FIR –filter? Explain why! (1p)
   b) Draw the approximate zero-pole diagram of the filter. (1p)
   c) According to zero-pole diagram what is the type of the filter (lowpass of highpass)? Explain why! (1p)

d) Calculate the impulse response of the filter (three coefficients are enough). (2p)

3. In a digital system the interesting band of a signal is in the range 0-30 kHz. The interesting band is going to be separated from the high frequency noise using FIR –filter. The other specifications of the filter are following.
   - Sampling frequency 200 kHz
   - Transition band width 10 kHz
   - Attenuation of the stopband 40 dB
   - Ripple of the passband 0.1 dB

   a) What window functions suit for the implementation of the filter when window –method is used? (1p)
   b) Calculate the length and the coefficients of the filter using window -method and most suitable window function (three coefficients is enough). (3p)

4. A digital signal contains interesting frequencies 10, 20 and 30 kHz with amplitudes 5, 3, 1. The sampling frequency is 100 kHz. The sampling frequency is lowered to 55 kHz.
   a) What frequency/frequencies will be aliased? (1p)
   b) Starting from the 2-order Butterworth filter (equation in the appendix) design an anti-aliasing filter using BZT –method. The 3 dB cutoff frequency is 20 kHz (sampling frequency is 100 kHz). Present the transfer function of the filter. (3p)
   c) Draw a diagram, which shows the steps of the decimation (lowering of the sampling frequency) when anti-aliasing filter is also used. (1p)
   d) Draw the amplitude spectrum of the decimated (including filtering) signal y(n) in the range [0, 55 kHz]. You can assume that filtering is done using the filter calculated in step-b or alternatively the filter $H(z) = 0.137\frac{1+z^{-1}}{1-0.7z^{-1}}$. Tell what filter you are using. (2p)

5. The transfer function of a filter used by a 16-bit system is

$$H(z) = \frac{0.15 + 0.20z^{-1} + 0.25z^{-2}}{1 + 0.10z^{-1} + 0.10z^{-2}}.$$

   The realization of the transfer function is implemented using so-called canonical second order section.
   a) Draw the realization diagram of the filter. (1p)

Table 1: Summary of important features of common window functions

Name of the window function	Transition width (Hz) Normalized	Pass-band ripple (dB)	Main lobe relative to side lobe (dB)	Stopband attenuation (max)	Window funktion, w(n), $\lvert n\rvert \le \frac{N-1}{2}$
Rectangular	0.9/N	0.7416	13	21	1
Hanning	3.1/N	0.0546	31	44	$0.5 + 0.5 \cdot \cos\frac{2\pi n}{N}$
Hamming	3.3/N	0.0194	41	53	$0.54 + 0.46\cos\frac{2\pi n}{N}$
Blackman	5.5/N	0.0017	57	74	$0.42 + 0.5\cos\frac{2\pi n}{N-1} + 0.08\cos\frac{4\pi n}{N-1}$
Kaiser	2.93/N, ($\beta = 4.54$) 4.32/N ($\beta = 6.76$) 5.71/N ($\beta = 8.96$)	0.0274 0.00275 0.000275		50 70 90	$\frac{I_0\left(\beta \cdot \left\{1 - \left[\frac{2n}{N-1}\right]^2\right\}^{1/2}\right)}{I_0(\beta)}$

Table 2: Summary of ideal impulse responses for standard frequency selective filters

Filter type	$h_D(n), n \ne 0$	$h_D(0)$
Lowpass	$2f_c \cdot \frac{\sin(n\omega_c)}{n\omega_c}$	$2f_c$
Highpass	$-2f_c \cdot \frac{\sin(n\omega_c)}{n\omega_c}$	$1 - 2f_c$
Bandpass	$2f_2 \cdot \frac{\sin(n\omega_2)}{n\omega_2} - 2f_1 \cdot \frac{\sin(n\omega_1)}{n\omega_1}$	$2(f_2 - f_1)$
Bandstop	$2f_1 \cdot \frac{\sin(n\omega_1)}{n\omega_1} - 2f_2 \cdot \frac{\sin(n\omega_2)}{n\omega_2}$	$1 - 2(f_2 - f_1)$

Butterworth-approximations:

n	Denominator (nimittäjä) of H(s)
1	$s + 1$
2	$s^2 + 1.414s + 1$
3	$(s^2 + s + 1)(s + 1)$

Numerator (osoittaja) of H(s) is always 1.