DIGITAL FILTERS 52337S
Exam 26.11.2002

YOU ARE ALLOWED TO BRING ONE A4-SIZE PAPER FILLED (BOTH SIDES CAN BE USED) WITH FORMULAS AND OTHER INFORMATION.

1. a) An analog signal is constructed from two sine waves with the same amplitude and with frequencies 10 and 30 Hz. The signal will be sampled with sampling frequency of 90 kHz. Sketch spectra of the original and sampled signals. Use the normalized frequency axis [0,1] for the sampled signal's spectrum. (1p)
   b) Determine a FIR-lowpass filter for the previous signal, using rectangular window. The passband edge frequency should be 10 kHz and stopband edge frequency should be 29 kHz. Calculate all coefficients. (2p)
   c) Filter the signal in 1.a) using the filter designed in 1.b). After that the sampling frequency will be lowered to 45 kHz. Sketch a spectrum of resulting signal using normalized frequency axis. (3p) (If you could not solve part b), use the following FIR-filter: $H(z)=0.19+0.2049z^{-1}+0.2101\,z^{-2}+0.2049\,z^{-3}+0.1900\,z^{-4}$. This has edge frequencies 11 and 30 kHz.)

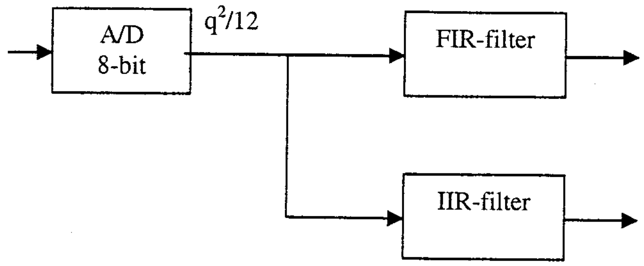
2. Calculate IDFT when DFT components are X(k) = {0 1 0 0 }. (3p)

3. a) Calculate the zero-pole diagram of the following filter. Determine and sketch the amplitude and phase response of the filter. (3p)
   b) Calculate the impulse response of the filter. Calculate 6 coefficients. (2p)

$$H(z)=\frac{0.566+0.566z^{-2}}{1+0.132z^{-2}}$$

4. A signal processing system should pass through only the interesting frequency 150 Hz. Your task is to determine a two-pole filter, which will block out other frequencies. The sampling frequency of the signal is 1 kHz, and 3 dB cut-off frequencies are 5 Hz below and above the interesting frequency. (5p)

5. When designing a signal processing system, the performances of FIR- and IIR-filters are compared with respect to finite wordlength effects. The processor here has 32-bit accumulator register. The original analog signal will be quantized to 8-bit, also the signal leaving the system will be 8-bit. You should use the IIR-filter in task 3 and corresponding six-point FIR-filter.



a) Draw the realizations for the filters with the numeric values of the coefficients, FIR-filter using the direct form and IIR-filter using second order canonic section. (2p)

b) Both filters are assumed to function without quantizing errors. Calculate the noise power of the signal at the outputs of the filters, taking in consideration possible roundings of intermediate results and A/D-conversion. Mark the noise sources to the diagrams in 5a). (3p)

Table 7.3 Summary of important features of common window functions.

Name of window function	*Transition width* (Hz) (*normalized*)	*Passband ripple* (dB)	*Main lobe relative to side lobe* (dB)	*Stopband attenuation* (dB) (*maximum*)	*Window function* $w(n), \lvert n\rvert \le (N-1)/2$
Rectangular	$0.9/N$	0.7416	13	21	1
Hanning	$3.1/N$	0.0546	31	44	$0.5 + 0.5\cos\left(\frac{2\pi n}{N}\right)$
Hamming	$3.3/N$	0.0194	41	53	$0.54 + 0.46\cos\left(\frac{2\pi n}{N}\right)$
Blackman	$5.5/N$	0.0017	57	75	$0.42 + 0.5\cos\left(\frac{2\pi n}{N-1}\right) + 0.08\cos\left(\frac{4\pi n}{N-1}\right)$
	$2.93/N\ (\beta = 4.54)$	0.0274		50	$\frac{I_0(\beta\{1-[2n/(N-1)]^2\}^{1/2})}{I_0(\beta)}$
Kaiser	$4.32/N\ (\beta = 6.76)$	0.002 75		70	
	$5.71/N\ (\beta = 8.96)$	0.000 275		90	

Table 7.2 Summary of ideal impulse responses for standard frequency selective filters.

Filter type	*Ideal impulse response*, $h_D(n)$: $h_D(n),\ n \neq 0$	$h_D(0)$
Lowpass	$2f_c \frac{\sin(n\omega_c)}{n\omega_c}$	$2f_c$
Highpass	$1 - 2f_c \frac{\sin(n\omega_c)}{n\omega_c}$	$1 - 2f_c$
Bandpass	$2f_2 \frac{\sin(n\omega_2)}{n\omega_2} - 2f_1 \frac{\sin(n\omega_1)}{n\omega_1}$	$2(f_2 - f_1)$
Bandstop	$1 - \left(2f_1 \frac{\sin(n\omega_1)}{n\omega_1} - 2f_2 \frac{\sin(n\omega_2)}{n\omega_2}\right)$	$1 - 2(f_2 - f_1)$

f_c, f_1 and f_2 are the normalized passband or stopband edge frequencies; N is the length of filter.