DIGITAL FILTERS 52337S
Exam 23.4.2002

1. The spectrum of the signal is composed of five different frequencies as shown in Figure 1. The spectrum is symmetric by zero; in the figure only positive frequencies are present.

a) What is the smallest sampling frequency that can be used without getting any aliasing errors? (1p)

b) Sketch the spectrum of the sampled signal in the range 0-10 kHz, when the original time domain signal is sampled at intervals of 0.1538 milliseconds. Is the signal aliased? (2p)

c) The original signal is bandlimited with second-order (n=2) Butterworth-filter, that have the following amplitude response:

$$|H(f)| = \sqrt{\frac{1}{1+\left(\frac{f}{f_c}\right)^{2n}}}$$

The low-pass filter has the cut-off frequency f_c = 2 kHz. Sketch the spectrum of the signal that is first filtered and then sampled at 0.1538 milliseconds. (2p)

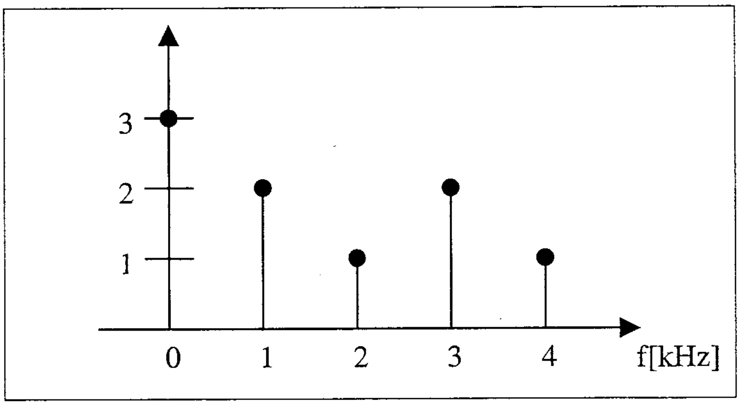


Figure 1.

2. A digital filter is characterized by the following transfer function:

$$H(z) = K\frac{1+z^{-2}}{1+r^2z^{-2}}$$

a) Calculate the zeros and poles of the filter and draw a zero-pole diagram. (1p)

b) Study the behaviour of the filter with different r values
   1) r = 0.9; 2) r = 0.5; 3) r = 0.1

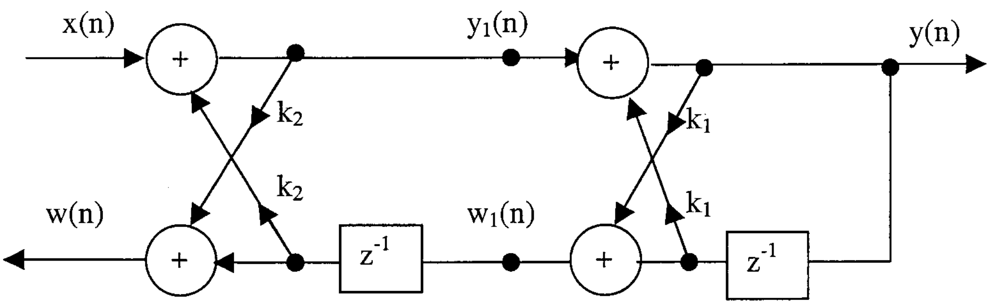
Sketch an amplitude response in all different cases. How the shape of the amplitude response is changed when parameter r is changed? (2p)

c) Scale the filter in the case 2 (r = 0.5) so that the maximum gain is one, say $\left|H\left(e^{j\omega T}\right)\right| = 1$, when r = 0.5. (2p)

3. A digital filter is characterized by the following transfer function:

$$H(z) = \frac{1}{1 + 0.1z^{-1} + 0.8z^{-2}}$$

a) Is this a FIR or IIR-filter? What type of filter is this (low-/highpass, bandpass/-stop)? (1p)
b) Determine the impulse response of the filter (4 values is enough). (2p)
c) The filter is realized with a lattice structure shown in Figure 2. Calculate the coefficients k_1 and k_2. (Notice that $x(n)$ is input and $y(n)$ is output) (2p)



4. A digital 8-bit system contains a lowpass filter, that has a following transfer function:

$$H(z) = \frac{1 + 2z^{-1} + z^{-2}}{1 - 0.099z^{-1} + 0.447z^{-2}}$$

a) Draw a realization diagram for the filter using second-order canonic section. (1p)
b) The calculations in the filter are handled with a processor that has 8-bit accumulator register. Mark clearly to the diagram in the assignment 4 a) the places where the roundings are made. (1p)
c) Determine the total noise power caused by the roundings to the filter output (assume that no scaling is used in this filter). (2p)

5. IIR-lowpass filter is characterized with the following transfer function:

$$H(z) = \frac{1 + 2z^{-1} + z^{-2}}{1 - 1.058z^{-1} + 0.338z^{-2}}$$

a) Draw a realization diagram using direct form noticing the scaling factor. The total gain of the system should stay the same regardless of the scaling. (2p)

b) Determine the scaling factors using L_1 and L_2-norms to prevent and minimize the probability of overflow. (2p)

c) Determine the **expression** for calculating the total noise power and explain what does the different terms in expression mean (you do not need to make the calculations). (1p)