DIGITAL FILTERS 52337S
Exam 11.1.2002

1. a) An analog input signal is bandlimited into 30 Hz by analog 3rd order Butterworth filter before signal is converted to digital. Determine the minimum sampling frequency F_s for the system, if aliasing error caused by sampling has to be smaller than 1 % of the signal level in the passband. (3p)

$$|H(f)| = \sqrt{\frac{1}{1+\left(\frac{f}{f_c}\right)^{2n}}}$$

   b) If signal is converted back to analog after processing, how large (in decibels) is the attenuation caused by sin(x)/x effect on the passband's edge? Assume that input signal was transferred to digital using ideal sampling and ADC, but it was reconstituted by using zero-order hold DAC. Sampling frequency is 128 Hz for the input and output. (2p)

2. a) Determine and plot a frequency and impulse responses for the following filter (at least 4 values has to be calculated). (4p)

$$H(z) = \frac{0.95 + 0.85z^{-2}}{1 + 0.75z^{-2}}$$

   b) Sketch a zero-pole-diagram for the filter. Is the filter stable based on the diagram? (1p)

3. Using the windowing method determine the coefficients (3 values is enough) of the highpass filter meeting the following specifications:
   - Passband starting from 9kHz
   - Transition width 1 kHz
   - Sampling frequency 25kHz
   - Stopband attenuation 48 dB

   Use the window function that is closest to the specifications. Which window functions could be used solving the problem? (5p)

4. A filter is characterized by the following transfer function:

$$H(z) = \frac{0.14 + +0.29z^{-1} + 0.14z^{-2}}{1 - 1.84z^{-1} + 0.95z^{-2}}$$

   The filter is realized in 8-bit system.

a) Draw a quantization noise model, when second-order canonic section is used to realize the filter. (2p)
b) Calculate the total output noise power for the filter. (2p)
c) Draw a quantization noise model and determine the equation of the total noise power, when the filter is realized with direct form. (2p)

5. Use the overlapp-add method to calculate the convolution of the sequences *x(n)*={2 0 1} and *y(n)*={1 3 2 3 -2 0 0 1 0 1 -1 2}. (3p)

Table 7.3 Summary of important features of common window functions.

Name of window function	*Transition width* (Hz) (*normalized*)	*Passband ripple* (dB)	*Main lobe relative to side lobe* (dB)	*Stopband attenuation* (dB) (*maximum*)	*Window function* $w(n), \lvert n \rvert \leq (N-1)/2$
Rectangular	$0.9/N$	0.7416	13	21	1
Hanning	$3.1/N$	0.0546	31	44	$0.5 + 0.5\cos\left(\frac{2\pi n}{N}\right)$
Hamming	$3.3/N$	0.0194	41	53	$0.54 + 0.46\cos\left(\frac{2\pi n}{N}\right)$
Blackman	$5.5/N$	0.0017	57	75	$0.42 + 0.5\cos\left(\frac{2\pi n}{N-1}\right) + 0.08\cos\left(\frac{4\pi n}{N-1}\right)$
Kaiser	$2.93/N\ (\beta = 4.54)$	0.0274		50	$\frac{I_0(\beta\{1-[2n/(N-1)]^2\}^{1/2})}{I_0(\beta)}$
	$4.32/N\ (\beta = 6.76)$	0.002 75		70	
	$5.71/N\ (\beta = 8.96)$	0.000 275		90	

Table 7.2 Summary of ideal impulse responses for standard frequency selective filters.

	Ideal impulse response, $h_D(n)$	
Filter type	$h_D(n), n \neq 0$	$h_D(0)$
Lowpass	$2f_c \frac{\sin(n\omega_c)}{n\omega_c}$	$2f_c$
Highpass	$-2f_c \frac{\sin(n\omega_c)}{n\omega_c}$	$1 - 2f_c$
Bandpass	$2f_2 \frac{\sin(n\omega_2)}{n\omega_2} - 2f_1 \frac{\sin(n\omega_1)}{n\omega_1}$	$2(f_2 - f_1)$
Bandstop	$2f_1 \frac{\sin(n\omega_1)}{n\omega_1} - 2f_2 \frac{\sin(n\omega_2)}{n\omega_2}$	$1 - 2(f_2 - f_1)$

f_c, f_1 and f_2 are the normalized passband or stopband edge frequencies; N is the length of filter.