Tekniikan matematiikka

DIFFERENTIAALIYHTÄLÖT (031076P)

2. välikoe, 2.5.2022

Lue tehtävät huolellisesti. Laskut, välivaiheet ja perustelut näkyviin. Pelkkä oikea vastaus ilman välivaiheita ei oikeuta täysiin pisteisiin.

1. Ratkaise alkuarvotehtävä

$$y'' + 4y' - 5y = 8\,\mathrm{e}^{-x},$$
$$y(0) = -1, y'(0) = 7.$$

2. Ratkaise Laplace-muunnoksella alkuarvotehtävä

$$y'' - 4y' + 4y = \delta(t-2)$$
$$y(0) = 0, y'(0) = 0.$$

3. Ratkaise differentiaaliyhtälösysteemin

$$\begin{cases} x'(t) = -x(t) + 2y(t), \\ y'(t) = 2x(t) + y(t) \end{cases}$$

yleinen ratkaisu ja se ratkaisupari, joka toteuttaa ehdot

$$x(0) = 0$$
$$y(0) = \sqrt{5}.$$

4. Määrää kaikilla parametrin $0 < a,\ a \neq 1$, arvoilla alkuarvotehtävän

$$\begin{cases} y'' + (1-2a)y' - 2ay = 6\,\mathrm{e}^{2x}, \\ y(0) = y'(0) = 0 \end{cases}$$

ratkaisu $y_a(x)$.

Laske raja-arvo

$$u(x) = \lim_{a \to 1} y_a(x)$$

kaikilla $x \in \mathbb{R}$.

Laplace-muunnoksen kaavoja:

$f(t)$	$F(s)$
1	$\frac{1}{s}$
$\frac{t^{n-1}}{(n-1)!}$	$\frac{1}{s^n}$
$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
e^{at}	$\frac{1}{s-a}$
$\frac{1}{(n-1)!}t^{n-1}\mathrm{e}^{at}$	$\frac{1}{(s-a)^n}$

$$\mathcal{L}(\mathrm{e}^{ct}f(t)) = F(s-c)$$
$$\mathcal{L}(t^n f(t)) = (-1)^n F^{(n)}(s)$$
$$\mathcal{L}(H(t-c)f(t-c)) = \mathrm{e}^{-cs}F(s)$$
$$\mathcal{L}(\delta(t-c)) = \mathrm{e}^{-cs}$$
$$\mathcal{L}(y'') = s^2Y(s) - sy(0) - y'(0)$$
$$\mathcal{L}(y') = sY(s) - y(0)$$
$$\mathcal{L}(\textstyle\int_0^t y(t)\mathrm{d}t) = \frac{1}{s}Y(s)$$

Differentiaaliyhtälöt: kaavat

x	$\cos x$	$\sin x$
$\frac{\pi}{6}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$
$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$
$\frac{\pi}{3}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$

$$\sin^2 x + \cos^2 x = 1 \qquad \tan x = \frac{\sin x}{\cos x} \qquad \cot x = \frac{1}{\tan x}$$

$$\sin x = -\sin(-x) = \cos\left(\frac{\pi}{2} - x\right) \qquad \cos x = \cos(-x) = \sin\left(\frac{\pi}{2} - x\right)$$

$$\sin 2x = 2\sin x \cos x \qquad \cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$$

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma} \qquad a^2 = b^2 + c^2 - 2bc\cos\alpha$$

$$\sin(x+y) = \sin x\cos y + \cos x \sin y \qquad \cos(x+y) = \cos x \cos y - \sin x \sin y$$

$$\sinh x = \frac{1}{2}(\mathrm{e}^x - \mathrm{e}^{-x}) \qquad \cosh x = \frac{1}{2}(\mathrm{e}^x + \mathrm{e}^{-x}) \qquad \tanh x = \frac{\sinh x}{\cosh x} \qquad \coth x = \frac{\cosh x}{\sinh x}$$

$$\cosh^2 x - \sinh^2 x = 1$$

$$Dx^n = nx^{n-1} \qquad D\sin x = \cos x \qquad D\cos x = -\sin x \qquad D\tan x = \frac{1}{\cos^2 x} = 1 + \tan^2 x$$

$$D\,\mathrm{e}^x = \mathrm{e}^x \qquad Da^x = a^x \ln a (a > 0) \qquad D\ln|x| = \frac{1}{x} \qquad D\log_a|x| = \frac{1}{x\ln a}(a > 0, a \neq 1)$$

$$D\overline{\text{arc}}\sin x = \frac{1}{\sqrt{1-x^2}} \qquad D\overline{\text{arc}}\cos x = -\frac{1}{\sqrt{1-x^2}} \qquad D\overline{\text{arc}}\tan x = \frac{1}{1+x^2}$$

$$(f^{-1})'(y_0) = \frac{1}{f'(x_0)}, \text{ missä } y_0 = f(x_0)$$

$$\int x^n\,\mathrm{d}x = \frac{x^{n+1}}{n+1} + C\ (n \neq -1) \qquad \int \frac{1}{x}\,\mathrm{d}x = \ln|x| + C \qquad \int \tan x\,\mathrm{d}x = -\ln|\cos x| + C$$

$$\int \frac{\mathrm{d}x}{\cos^2 x} = \int (1 + \tan^2 x)\,\mathrm{d}x = \tan x + C \qquad \int \frac{\mathrm{d}x}{\sin^2 x} = \int (1 + \cot^2 x)\,\mathrm{d}x = -\cot x + C$$

$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \overline{\text{arc}}\ \sin x + C \qquad \int \frac{\mathrm{d}x}{1+x^2} = \overline{\text{arc}}\ \tan x + C$$

$$A = \int_a^b |f(x)|\,\mathrm{d}x \qquad V = \pi\int_a^b (f(x))^2\,\mathrm{d}x$$

$$\begin{aligned} Q(x) &= (x-x_1)^{k_1}\ldots(x-x_r)^{k_r}(x^2+c_1x+d_1)^{l_1}\ldots(x^2+c_sx+d_s)^{l_s}; \\ \frac{P(x)}{Q(x)} &= \frac{A_{1,1}}{x-x_1} + \ldots + \frac{A_{1,k_1}}{(x-x_1)^{k_1}} + \ldots + \frac{A_{r,1}}{x-x_r} + \ldots + \frac{A_{r,k_r}}{(x-x_r)^{k_r}} \\ &+ \frac{B_{1,1}x + C_{1,1}}{x^2+c_1x+d_1} + \ldots + \frac{B_{1,l_1}x + C_{1,l_1}}{(x^2+c_1x+d_1)^{l_1}} + \ldots \\ &+ \frac{B_{s,1}x + C_{s,1}}{x^2+c_sx+d_s} + \ldots + \frac{B_{s,l_s}x + C_{s,l_s}}{(x^2+c_sx+d_s)^{l_s}} \end{aligned}$$