**Artificial Intelligence (521495A), Spring 2022**
**Exercise 2 : Probability and Bayesian networks**
**Deadline for reports: Wed 2.2.2022 23:59 (+1h)**

This handout contains five problems related to probability and Bayesian networks lectures. Problems 1-4 are pre-exercises to support learning and solutions to them are provided in **solutions2.pdf**. For Problem 5, the solution is not given and you should return a solution as a report. **Include in the report only the solution to Problem 5**. The solution gives max. 1 point, which is taken into account in course grading.

Problems 1-5: Background material is provided in Lectures 3 and 5 (Track 2). See also the course book Chapters 13 and 14.

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**Problem 1**. Let's have a joint probability table of variables "well-prepared" (W, +w: yes, -w: no) and "passing the exam" (E, +e: yes, -e: no), when studying students' success in the course.

| $W$ | $E$ | $P(W,E)$ |
|---|---|---|
| +w | +e | 0.45 |
| +w | -e | 0.10 |
| -w | +e | 0.05 |
| -w | -e | 0.40 |

Calculate the following conditional probabilities

(a) $P(+w \mid +e)$

(b) $P(-w \mid +e)$

(c) $P(+e \mid -w)$

(d) $P(W \mid +e)$ using normalization trick

**Problem 2**. A patient takes a cancer test and the result is positive. The test returns a correct positive 97% of the cases and correct negative 95% of the cases. Furthermore 0.004 of the population have the cancer.

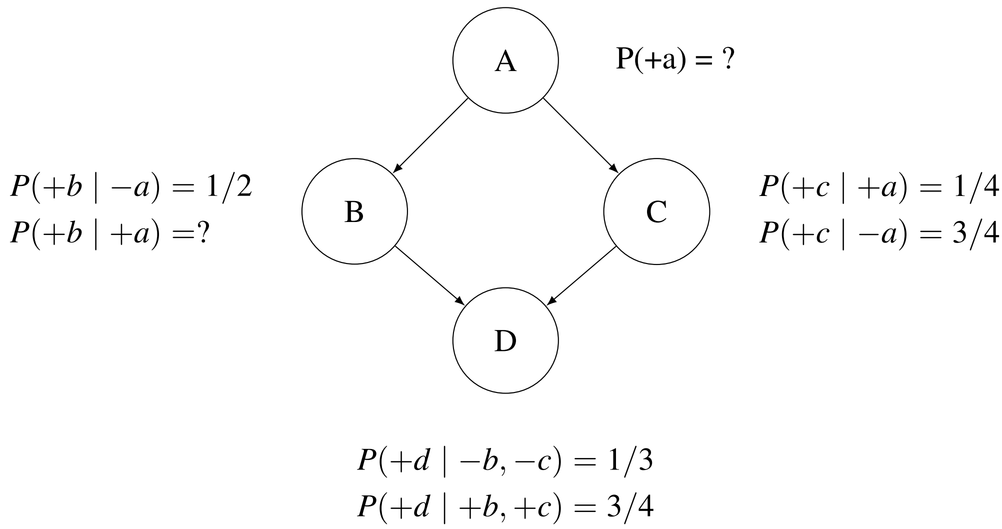
Does the patient have cancer or not? Use Bayes rule to find out.

**Problem 3**. Consider the Bayesian network below. Asssuming that $P(+a, +b, +c, +d) = 1/24$ and $P(-a, -b, -c, -d) = 1/36$.

Calculate the following probabilities

(a) $P(+a)$

(b) $P(+b \mid +a)$



**Problem 4**. Consider a Bayesian network for medical diagnosis where having heart problems (H, +h: yes, -h: no) depends on blood pressure (B, +b: high, -b: low). Furthermore, blood pressure depends on doing exercises (E, +e: yes, -e: no) and having healthy diet (D, +d: yes, -d: no). The probability tables are given below.

(a) Construct a Bayesian network for given problem

(b) Formulate the equation for joint probability distiribution $P(E,D,B,H)$ using the network structure (i.e., the conditional independencies)

(c) Calculate the probability of $P(+e,-d,+b,+h)$

| $E$ | $P(E)$ |
|---|---|
| +e | 0.5 |
| -e | 0.5 |

| $D$ | $P(D)$ |
|---|---|
| +d | 0.75 |
| -d | 0.25 |

| $E$ | $D$ | $B$ | $P(B \mid E,D)$ |
|---|---|---|---|
| +e | +d | +b | 0.05 |
| +e | +d | -b | 0.95 |
| +e | -d | +b | 0.55 |
| +e | -d | -b | 0.45 |
| -e | +d | +b | 0.45 |
| -e | +d | -b | 0.55 |
| -e | -d | +b | 0.90 |
| -e | -d | -b | 0.10 |

| $B$ | $H$ | $P(H \mid B)$ |
|---|---|---|
| +b | +h | 0.75 |
| +b | -h | 0.25 |
| -b | +h | 0.05 |
| -b | -h | 0.95 |

**Problem 5* [1p]**. Consider a Bayesian network for avalanche risk analysis where the occurence of avalanche (A, +a: yes, -a: no) depends on the steepness of the mountain area (S, +s: steep, -s: gently) and the snow conditions (C, +c: unstable, -c: stable). Furthermore, snow conditions depends on the recent weather (W, +w: unfavourable, -w: favourable), where unfavourable weather increases the unstability of snow conditions. The marginal and conditional probability tables are given below.

(a) Construct a Bayesian network for a given problem

(b) Calculate $P(A \mid +s)$, i.e., the probability distribution of avalanche occurence when skiing on a steep mountain area. Use variable elimination to perform the inference

| $S$ | $P(S)$ |
|---|---|
| +s | 0.3 |
| -s | 0.7 |

| $W$ | $P(W)$ |
|---|---|
| +w | 0.6 |
| -w | 0.4 |

| $S$ | $C$ | $A$ | $P(A \mid S,C)$ |
|---|---|---|---|
| +s | +c | +a | 0.80 |
| +s | +c | -a | 0.20 |
| +s | -c | +a | 0.30 |
| +s | -c | -a | 0.70 |
| -s | +c | +a | 0.20 |
| -s | +c | -a | 0.80 |
| -s | -c | +a | 0.01 |
| -s | -c | -a | 0.99 |

| $W$ | $C$ | $P(C \mid W)$ |
|---|---|---|
| +w | +c | 0.8 |
| +w | -c | 0.2 |
| -w | +c | 0.4 |
| -w | -c | 0.6 |